COMM 225 WAITING LINE (CHAPTER # 18) QUESTIONS FOR PRACTICE WITH SOLUTIONS

Q1 (Ref. Q# 2, page 748 of the Text Book.)

Repair calls for Xerox copiers in a small city are handled by one repair person. Repair duration, including travel time, is exponentially distributed with a mean of two hours per call. Requests for copier repairs come in at a mean rate of three per eight-hour day (assume Poisson). Assume infinite source. Determine: LO4

a. **The average number of copiers waiting for repair.**
b. **Server utilization.**
c. **The amount of time during an eight-hour day that the repair person is not out on a call.**
d. **The probability of two or more copiers in the system (waiting to be or being repaired).**
e. **The probability that a copier waits more than four hours for repair to begin.**

Q2(Ref. Q# 21, page 751 of the Text Book.)

The number of customers coming to a bank between 1:00 p.m. and 1:30 p.m. of a "normal" day has a Poisson distribution with an average of 39 customers during the half hour. The length of time each customer spends with a teller is exponential with an average of 45.5 seconds. The average time a customer spends in the line waiting is desired to be three minutes. The manager is wondering how many tellers are required during this time period. LO5

Q3 (Ref. Q# 27, page 751 of the Text Book.)

The Model 8300 Telemetry system used at Wesley Long Community Hospital, Greensboro, North Carolina, is a completely self-contained, wireless, one-patient cardiac monitor.[6] The system provides wireless transmission of a patient's heartbeat to a receiver either at a central station or at the bedside. A study was undertaken to analyze the service provided by the 18 currently held telemetry units and to investigate the cost/benefit of any additional units. During a 38-day reviewing period, there were 156 requests for service (telemetry units are requested and used 24 hours a day). The average service duration was 93.6 hours per patient. Both interarrival and service durations had exponential distributions. LO5

a. **What is the average arrival rate per day of patients needing telemetry?**
b. **What is the average service rate per day of patients needing telemetry?**
c. **If the average wait time before service is desired to be less than 5 hours, are 18 telemetry units enough? (*Hint:* For λ/μ = 16 and M = 18, it can be shown that L_q = 4.29.)**

Q4 (Ref. Q# 23, page 751 of the Text Book.)

The number of calls coming into a child abuse hotline during the 5:00 p.m. to 9:00 a.m. period on weekdays had a Poisson distribution with an average of 1.67 calls per hour. Call lengths had exponential distribution with an average of 5.4 minutes. It is considered important that a very high percentage of calls get through to the operator right away (as opposed to having to wait). If only one person is assigned to answer the phones, what is the probability that a call will have to wait because the operator is busy? LO4

Q5 (Ref. Q# 30, page 752 of the Text Book.)

The Columbia University Hospital wanted to know if it needed to have more than one operating room (OR) crew present during the night shift (11:00 p.m. to 7:00 a.m.).[14] If the probability that one crew is busy is greater than 1 Page 753 percent, then a second OR crew would be required. During one year, 62 patients required emergency operations during the night shift. The average operation took 80.79 minutes. Does the hospital need a second OR crew during the night shift? LO4

SOLUTIONS

Q1. [Single server, Exponential service durations]

Average repair time = 2 hours

$\lambda = 3/\text{day}$

8 hours a day

μ = 8 hours a day / 2 hours per call = 4 calls /day

a. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{3^2}{4(4-3)} = 2.25$ copiers

b. $\rho = \dfrac{\lambda}{\mu} = \dfrac{3}{4} = .75$, or 75%

c. Proportion of time idle $P_o = 1 - \rho = 1 - .75 = .25$.
Thus, average hours idle per day = .25(8 hours per day) = 2 hours.

d. $P(L_s \geq 2) = P_{n \geq 2} = 1 - P_{n<2}$

$$= 1 - \left(1 - \left(\frac{\lambda}{\mu}\right)^2\right) = \left(\frac{\lambda}{\mu}\right)^2$$

$$= \left(\frac{3}{4}\right)^2 = .5625$$

e. $P_{W\,inQ>4hours} = \dfrac{\lambda}{\mu} e^{-(\mu-\lambda)(0.5day)} = \dfrac{3}{4} e^{-(4-3)(0.5day)} = .455$

Q2. [Multiple servers, Exponential service durations]

λ = 39/half hour = 1.3 customers/min.

Avg. service time = 45.5 sec. → μ = 1.32 round to 1.3 customers/min.

W_q should be = 3 min.

M?

$\lambda / \mu = 1.3 / 1.3 = 1$

From Table 18-4

M	L_q	$W_q = L_q / \lambda$
1	--	--
2	.333	.333/1.3 = .26 min = 15 sec.

Because 15 sec < 3 min., 2 tellers are adequate.

Q3. [Multiple servers, Exponential service durations]

a. $\lambda = 156 / 38 = 4.1$ requests/day

b. Avg. service duration = 93.6 hours → $\mu = 24 / 93.6 = 0.2564$ requests/day

c. W_q should be < 5 hours = .21 day
Is M = 18 adequate?

$\lambda / \mu = 4.1/0.2564 = 15.99 \sim 16$

It can be shown that for $\lambda / \mu = 16$ and $M = 18$, $L_q = 4.29$

$W_q = L_q / \lambda = 4.29/4.1 = 1.05$ day = 25 hrs > 5 hrs.

Therefore, M = 18 is not enough.

Q4. [Single server, Exponential service durations]

$\lambda = 1.67$ calls / hr.

Avg, service duration = 5.4 min / call → $\mu = 60 / 5.4 = 11.11$ calls / hr

M = 1

Prob wait?

Prob. wait = $1 - P_0 = 1 - (1 - \lambda / \mu) = \lambda / \mu = 1.67 / 11.11 = .15$ or 15%

Q5. $\lambda = 62$ per year = $62 / 365 = .17$ per night shift

Avg service time = 80.79 min → $\mu = 60(8 \text{ hrs per shift}) / 80.79 = 5.94$ per shift

Prob busy = $1 - P_0 = \lambda / \mu = .17 / 5.94 = .0286 > .01$ → need a 2nd shift