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CHG 2314

March 6, 2019

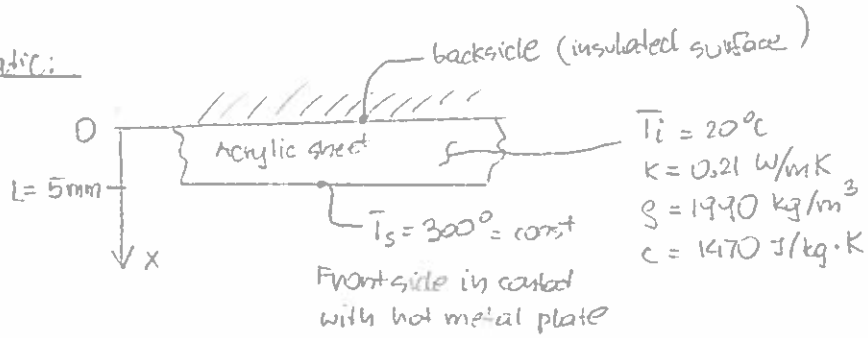
Quiz 7

Solution

Consider an acrylic sheet of thickness L = 5 mm that is used to coat a hot isothermal metal substrate at T_h = 300°C. The properties of the acrylic are ρ = 1990 kg/m^3, c = 1470 J/kg K, and k = 0.21 W/m K. The initial temperature of the acrylic sheet is T_i = 20°C.

a) If upon the contact with the hot metal substrate the surface temperature of acrylic becomes instantaneously T_s = 300°C, what is the equivalent Biot number for the acrylic sheet? **(2 points)**
b) Assuming that the backside of the acrylic sheet is perfectly insulated, how long it would take backside of the acrylic sheet temperature to reach 100°C? **(8 points)**

Schematic:



Assumptions: (1) 1-D conduction, (2) One-term approximation valid i.e. $Fo > 0.2$, (3) constant prop.

a) $Bi = \frac{L_c h}{k}$ where $L_c = 0.005$ m, h is not given but since T_s becomes instantaneously constant, it is equivalent to $h \to \infty$

$\therefore Bi \to \infty$

b) Time for the backside ($x = 0$) to reach 100°C ie $T(x=0, t) = 100°C$ where $t = ?$

- For plane wall and $Bi = \infty$: $\zeta_1 = 1.5708$, $C_1 = 1.2733$

$$\frac{T(x=0,t) - T_s}{T_i - T_s} = C_1 \exp(-\zeta_1^2 Fo) \Leftrightarrow \frac{100-300}{20-300} = 0.71429 = 1.2733 \exp(-1.5708^2 \cdot Fo)$$

Rearranging: $Fo = \ln\left(\frac{0.71429}{1.2733}\right) / (-1.5708^2) = 0.2343 > 0.20 \rightarrow$ ok!

- Definition of $Fo = \frac{t\alpha}{L^2}$ where $\alpha = \frac{k}{\rho c} = \frac{0.21}{1990 \cdot 1470} = 7.18 \cdot 10^{-8} \frac{m^2}{s}$

$$\Rightarrow t = \frac{Fo \cdot L^2}{\alpha} = \frac{0.2343 \cdot (0.005)^2}{7.18 \cdot 10^{-8}} = 81.6 \text{ s}$$