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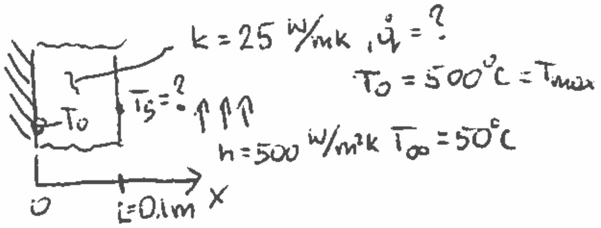
Quiz 4

A plane wall of thickness 0.1 m and thermal conductivity 25 W/m K having uniform volumetric heat generation $\dot{q}$ is perfectly insulated on one side, while the other side is exposed to a fluid at T_∞ = 50ºC. The convection heat transfer coefficient between the wall and the fluid is h = 500 W/m² K.

If the maximum temperature of the wall, which occurs at the insulated surface is T_0, 500°C determine the exposed surface temperature of the wall (T_s) and volumetric heat generation ($\dot{q}$). <u>Please note you do not need to derive the equation for the temperature distribution in the wall, you may choose one from among the equations at the back of the page</u>.

Schematic:



Assumptions:

1) 1-D conduction in x-direction
2) Steady-state
3) Constant properties
4) Negligible radiation

Analysis

- Temperature profile is given by: $T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$ which at $x=0$ becomes

$T_0 = \frac{\dot{q}L^2}{2k} + T_s$ (1) There are 2 unknowns in Eq(1) $\dot{q}$ and T_s. we need another independent equation

- Energy balance on the wall:

$\dot{q}\cancel{A_s}L = \cancel{A_s}h(T_s - T_\infty) \Rightarrow \dot{q}L = h(T_s - T_\infty)$ (2) second independent equation

From (1): $T_s = T_0 - \frac{\dot{q}L^2}{2k} \Rightarrow$ Sub-in into (2): $\dot{q}L = h\left(T_0 - \frac{\dot{q}L^2}{2k} - T_\infty\right)$ ... (3)

- Rearranging Eq (3) $\dot{q}L + \dot{q}\frac{L^2h}{2k} = h(T_0 - T_\infty) \Rightarrow \dot{q} = \left[\frac{h(T_0 - T_\infty)}{L + \frac{L^2h}{2k}}\right]$ ... (4)

Sub-in numerical values:

$\dot{q} = \frac{500(500-50)}{\left[0.1 + \frac{0.1^2 \cdot 500}{2 \cdot 25}\right]} = 1.125 \cdot 10^6 \frac{W}{m^3}$ ◀ $T_s = 500 - \frac{1.125 \cdot 10^6 \cdot 0.1^2}{2 \cdot 25} = 275°C$

units $\left[\frac{\frac{W}{m^2K} \cdot \cancel{K}}{m + \frac{m^2 W mK}{m^2 K W}} = \frac{W}{m^3} \text{ ok}\right]$ $\left[K - \frac{\frac{W}{m^3} \cdot m^2}{\frac{W}{mK}} = K \text{ ok}\right]$