Conservation of Angular Momentum

TM: **P** = m**v**; **Fnet** =**ΔP**/Δt ; If $\vec{F}$net = 0, then **ΔP** = 0, then, **P** = a constant, therefore, Conservation of Liner Momentum, **P.**

In the same manner:

RM: $\vec{L} = I\vec{\omega}$, $\vec{\Gamma}_{net} = \Delta L/\Delta t$; If $\vec{\Gamma}_{net} = 0$, then $\Delta L = 0$, **L** = a constant, therefore conservation of Angular Momentum.

Some examples:

1. A disk (cylindrical) of mass of 10 kg and radius of 50 cm is spinning freely at 3 rads/s. A second identical disk slides down a spindle and they rotate together. (a) what is the angular velocity of the combination? (b) What is the change in kinetic energy of the system?
2. A person is on platform that rotates at 0.5 revs/s. With arms outstretched s(he) holds two 4-kg blocks at a distance of 1 m from the axis of rotation-which passes through her/him. S(he) then reduces the distance of the blocks from the axis to 0.5 m. Assume the mass and the radius of the platform to be 10 kg and 50 cm, respectively. Approximate the person's body to be cylindrical and the arms of negligible mass. Take the person's mass to be 60 kg and her/his radius to be 30 cm. (a) What is the new angular velocity? (b) What is the change in kinetic energy?
3. A person of mass, m, of 80 kg runs at a speed of 4 m/s along the tangent to a disk-shaped platform of mass, M, 160 kg and radius R = 2 m. The platform is initially at rest but can rotate freely about an axis through its center. (a) Find the angular velocity of the platform after the person jumps on. (b) S(he) then walks to the center. Find the new angular velocity. Treat the person as a point particle.

NYA(S12)

Consv of Ang Mom solutions

Ex 1. Disk 1: $m_1 = 10 kg$, $R_1 = 0.50 m$, $\omega_{10} = 3$ rads/s

Disk 2: Identical : $I_1 = I_2$, $\omega_{20} = \phi$

(a) $\omega_{f_{12}} = ?$ (b) $\Delta KE_{syst} = ?$

(a) $\vec{\Gamma}_{\substack{net\\ext}} = \frac{\Delta \vec{L}}{\Delta t}$; where $\vec{\Gamma}_{\substack{net\\ext}} = 0 \Rightarrow \Delta \vec{L} = 0 \Rightarrow \vec{L} = $ cnst

$\vec{l}_{10} + \vec{l}_{20} = \vec{l}_{12f} \Rightarrow I_1 \vec{\omega}_{10} + I_2 \cancel{\vec{\omega}_{20}}^{0} = (I_1 + I_2)\vec{\omega}_f$

where, $I_1 = I_2 = I$ $\Rightarrow \cancel{I}(3) = (2\cancel{I})\vec{\omega}_f \Rightarrow \omega_f = 1.5 \frac{rads}{s}$

same dir^n as $\vec{\omega}_{10}$.

(b) $\Delta K = K_f - K_0 = \frac{1}{2}(I_1 + I_2)\omega_f^2 - \frac{1}{2} I_1 \omega_0^2 = 0$

where, $I_1 = I_2 = \frac{1}{2}(10)(0.5^2) = 1.25 \ kg.m^2$

$\Rightarrow \Delta K = \frac{1}{2}(2.5)(1.5^2) - \frac{1}{2}(1.25)(3^2) = 2.81 - 5.62 = -2.8 J.$

Ex 2 $\omega_{pl_0} = 0.5$ rev/s , $m_{pl} = 10 kg$, $R_p = 0.50 m$

$2 \times 4 kg$ masses, $r_{m_0} = 1 m$ each

$r_{m_f} = 0.5 m$ each

$m_p = 60 kg$

$r_p = 0.30 m$

{(a) $\omega_f = ?$, (b) $\Delta K = ?$

system = (pl + p + 2xm)

$\vec{\Gamma}_{\substack{net\\ext\,syst}} = 0$

$\vec{l}_{pl_0} + \vec{l}_{p0} + 2\vec{l}_{m_0} = \vec{l}_{pl_f} + \vec{l}_{pf} + 2\vec{l}_{m_f}$

$\left[\underbrace{\tfrac{1}{2}(10)(0.5)^2}\right](0.5) + \left[\tfrac{1}{2}(60)(0.3^2)\right](0.5) + \left[2(4)(1^2)(0.5)\right] = (I_{pl} + I_p + 2I_{m_f})\vec{\omega}_f$

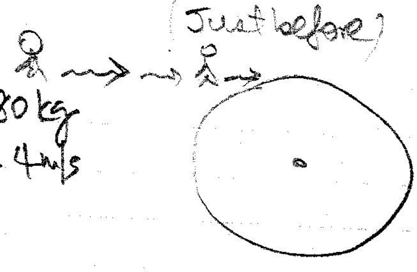
$(\underset{I_{pl}}{1.25} + \underset{I_p}{2.7} + \underset{2 \times I_{m_0}}{8})\underset{\omega_0}{(0.5)} = [1.25 + 2.7 + \underbrace{2(4)(0.5^2)}_{2}]\vec{\omega}_f$

$6.0 = 5.95 \omega_f$

$\Rightarrow \omega_f = 1$ rev/s same dir^n as $\vec{\omega}_0$

Ex 3.

(Just before)

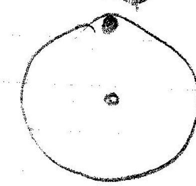


$M_p = 80\,kg$
$V_{p_0} = 4\,m/s$

$M_{pl} = 160\,kg$
$R_{pl} = 2\,m$
$\omega_{pl_0} = 0$

BEFORE } AFTER (Just After)

m_p



pl

(a) $\vec{\tau}_{net\,ext} = 0 = \frac{\Delta \vec{L}}{\Delta t} \Rightarrow \vec{L} = const \Big|$ system = (p + pl).

$$\vec{l}_{p_0} + \vec{l}_{pl_0} = \vec{l}_{p,pl\,f}$$

$$\underbrace{I_p \vec{\omega}_{p_0}}_{r \times P} + I_{pl} \cancel{\vec{\omega}_{pl_0}}^{0} = (I_p + I_{pl})\vec{\omega}_f \quad \Big\} \text{ where, } I_p = m r^2, \quad I_{pl} = \frac{MR^2}{2}$$

$$m_p V_{p_0} r_p + 0 = \left[m_p r_p^2 + \frac{m_{pl} R_{pl}^2}{2}\right]\vec{\omega}_f$$

$$(80)(4)(2) = \left[(80 \times 2^2) + \frac{160(2)^2}{2}\right]\vec{\omega}_f$$

$$640 = [320 + 320]\vec{\omega}_f \Rightarrow \vec{\omega}_f = 1\,\frac{rad}{s}\ \text{cw}$$

(b) when the person is at the center,

$\Rightarrow r_p = 0 \Rightarrow I_p = m_p r_p^2 = 0$

$\Rightarrow \vec{L}_0 = \vec{L}_f = \vec{L}_{f'}$ (center)

$\Rightarrow \vec{L}_0 = \vec{L}_{f'}$

$$640 = I_{pl}\,\vec{\omega}_f' \Rightarrow 640 = 320\,\vec{\omega}_f' \Rightarrow \vec{\omega}_f' = 2\,rad/s\ \text{cw}$$