203-NYA(A13) - 2D Collision - Another Example

$m_1 = 1000\,kg$; $\vec{v}_{1i} = 50$ km/hr North; $|\vec{v}_{1f}| = 24$ km/hr, $\boxed{\theta_{1f} = ?}$

$m_2 = 3000\,kg$; $\vec{v}_{2i} = 70$ km/hr 50° NE; $\boxed{v_{2f} = ?}$, $\theta_{2f} = 55°$ NE

NOTE: • Keep the unit for "v" in km/hr, v_{2f} will be in km/hr also.
• Factorize both sides by 10^3 to simplify the math.

• Collision: system $= (m_1 + m_2)$: $\vec{F}^{\,\nearrow 0}_{net\ ext\ system} = \dfrac{\Delta \vec{P}^{\,\nearrow 0}_{syst}}{\Delta t}$

$\Rightarrow \vec{P}_{syst} = cnst \Rightarrow \vec{P}_{syst_{JBC}} = \vec{P}_{syst_{JAC}}$: $\vec{P}$ is conserved

$\Rightarrow [1(50\hat{j}) + 3(70\cos 50°\,\hat{i} + 70\sin 50°\,\hat{j})] \times 10^3 = [1(24\cos\theta_{1f}\,\hat{i} + 24\sin\theta_{1f}\,\hat{j}) + 3(v_{2f}\cos 55°\,\hat{i} + v_{2f}\sin 55°\,\hat{j})] \times 10^3$

$\Rightarrow (50\hat{j} + 135\hat{i} + 161\hat{j}) = (24\cos\theta_{1f}\,\hat{i} + 24\sin\theta_{1f}\,\hat{j} + 1.72\,v_{2f}\,\hat{i} + 2.46\,v_{2f}\,\hat{j})$

$\Rightarrow$ x-comps: $135 = 24\cos\theta_{1f} + 1.72\,v_{2f}$ — (I)

y-comps: $211 = 24\sin\theta_{1f} + 2.46\,v_{2f}$ — (II)

Solve (I) for v_{2f}: $v_{2f} = \dfrac{135 - 24\cos\theta_{1f}}{1.72} = (78.49 - 13.95\cos\theta_{1f})$

Subst v_{2f} into (II): $211 = 24\sin\theta_{1f} + 2.46(78.49 - 13.95\cos\theta_{1f})$

$\Rightarrow 211 = 24\sin\theta_{1f} + 193.1 - 34.3\cos\theta_{1f}$

$\Rightarrow 24\sin\theta_{1f} - 34.3\cos\theta_{1f} - 17.9 = 0$; where, $\cos\theta_{1f} = \sqrt{1 - \sin^2\theta_{1f}}$

$\Rightarrow 24\sin\theta_{1f} - 34.3\sqrt{1 - \sin^2\theta_{1f}} - 17.9 = 0$: let $x = \sin\theta_{1f}$

$\Rightarrow 0.70x - 0.52 = \sqrt{1 - x^2}$: square both sides and use $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow 1.49x^2 - 0.73x - 0.73 = 0$ Quadratic Eqn

$$\Rightarrow X_{12} = \sin\theta_{1f} = \frac{0.73 \pm \sqrt{(0.73)^2 - 4(1.49)(-0.73)}}{2(1.49)} \quad \left\{ \because \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right.$$

$$\left. \because a = 1.49,\ b = -0.73,\ c = -0.73 \right\}$$

$$\Rightarrow (\sin\theta_{1f})_1 = 0.997 \Rightarrow \boxed{\theta_{1f} = 86^\circ}$$

$(\sin\theta_{1f})_2 = -0.5$, but makes no sense physically

$\Rightarrow$ Subst. into v_{2f}: $v_{2f} = 78.49 - 13.95\cos 86^\circ$

$$= 78.49 - 0.97$$

$$\boxed{v_{f_2} = 77.5 \text{ km/hr}}$$