KINEMATICS-Analytical Method

In cases where acceleration remains a constant in an interval (that means, the net force, that generates the acceleration, remains a constant in an interval, one can use the two definitions and the five derivations to find the solution to a given problem, no matter how complicated.

In this regard, it helps to categorize the problems into the following different categories:

- **One Object, one interval (with a constant acceleration)**
  Try the following two examples in this category:
  (1). If an object could accelerate continuously at 10 m/s^2, how far would it travel and how long would it take to reach (a) the speed of sound, 330 m/s, (b) the excape speed of a rocket from the earth, 11.2 km/s?
  (2). An object is thrown vertically upward at 30 m/s, from the roof top of a building of height of 300 m. Find: (a) its position and velocity 3 seconds after thrown, (b) the velocity with which it hits the ground, as well as the total time of flight, (c) the time to reach the maximum height and the maximum height.

- **Two objects, one interval, starting from the same point, at the same time**
  Ex. A car passes a traffic light at 100 Km/hr and continues at a constant velocity. A police car, waiting at the traffic light, accelerates from rest at 2 m/s^2. (a) Where and when does the police catch the truck? (b) What is its velocity then?
- **Two objects, one interval, starting at different points and at different times.**
  Ex. An object (A) is thrown vertically upward at 30 m/s, from the roof top of a building of height of 300 m. Four seconds later, another object (B) is thrown vertically upward, from the ground, at 100 m/s. (a) Find where and when, if at all, do A and B cross? (b) What are their velocities then?
- **One Object, multiple intervals (with different accelerations)**
  Ex. The driver of a truck moving at 30 m/s suddenly notices a moose 70 m straight ahead. If the driver's reaction time is 0.5 s and the maximum deceleration is 8 m/s^2, can the driver avoid hitting the moose without steering to one side?

1D KINS: Analytical

(A1)

• One obj, one interval (const "a")

① $a = 10\ m/s^2$; $\Delta x, t = ?$ (Assuming it starts from rest, $v_0 = 0$)

(a) $v_f = 330$ m/s

Use: $v_f^2 = v_0^2 + 2a\Delta x \Rightarrow 330^2 = 0^2 + 2(10)(\Delta x) \Rightarrow \Delta x = 5,445\ m$

Use: $v_f = v_0 + at \Rightarrow 330 = 0 + 10(t) \Rightarrow t = 33\ s$

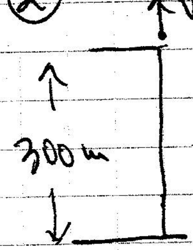
To check if $\Delta x \stackrel{?}{=} \left(\frac{v_0 + v_f}{2}\right)t$ ie. $5445 \stackrel{?}{=} \left(\frac{0+330}{2}\right)(33)$

$5445 \stackrel{\checkmark}{=} 5445$ ✓

(Repeat)
(b) Try the same thing for $v_f = 11.2$ km/s $= 11.2 \times 10^3$ m/s $= 11,200$ m/s.

⋮

② $v_0 = 30$ m/s   (a) $y, v_f = ?$ $\Delta t = 3s$



three givens in the interval. $a = -10\ m/s^2$, $v_0 = +30\ m/s$, $t = 3s$.

Use: $y = y_0 + v_0t + \frac{1}{2}at^2 = 0 + 30(3) - 5(3^2) = 90 - 45 = +45\ m$

ie. 45 m above the roof top

or 345 m " " ground.

$v_f = v_3 = ?$: Use: $v_f = v_0 + at = 30 - 10(3) = \emptyset$ ie. it is at y_{max} ∴ $V = 0$ temporarily.

(b) (i) $v_f = ?$ Use: $v_f^2 = v_0^2 + 2a\Delta y$

Δy = Δ in position = Displ. = −300 m
$a = -10\ m/s^2$
$v_0 = +30\ m/s$

$\Rightarrow v_f^2 = 30^2 - 20(-300)$
$= 900 + 6000 \Rightarrow v_f = \pm\sqrt{6900} = -83\ m/s$ on the way down.

(ii) $t = ?$ Use: $\Delta y = v_0t + \frac{1}{2}at^2 \Rightarrow -300 = 30t - 5t^2 \Rightarrow$ solve for t.

or use: $v_f = v_0 + at \Rightarrow -83 = 30 - 10t \Rightarrow t = 11.3\ s$

(c) $y_{max} = ?$ and $t_{y_{max}} = ?$ (A2)

3 Givens: $a = -10\ m/s^2$, $v_0 = +30\ m/s$, $v_f = 0$.

(i) $y_{max} = ?$

Use: $v_f^2 = v_0^2 + 2a\Delta y \Rightarrow 0 = 30^2 - 20\,\Delta y_{max}$

$\Rightarrow \Delta y_{max} = \frac{900}{20} = \underline{45m}$

(ii) $t = ?$: Use: $v_f = v_0 + at \Rightarrow 0 = 30 - 10t$

$\Rightarrow \underline{t = 3\,s}$ to reac y_{max}

→ Two objs, one interval, starting from the same pt. (ie. $y_{10} = y_{20} = 0$) at the same time (ie. $t_1 = t_2$).

Use the Explicite form: $y = y_0 + v_0 t + \frac{1}{2}at^2$

rather than $\Delta y = v_0 t + \frac{1}{2}at^2$

Ex. $v_{c0} = 100\ km/hr = (100/3.6)\ m/s = 27.8\ m/s = \text{cnst}$.

$a_{pc} = 2\ m/s^2$, $v_{0pc} = 0$

(a) $x_1 = x_2 = ?$ $t_1 = t_2 = ?$

Use: $\overbrace{(\cancel{x_0} + v_0 t + \cancel{\tfrac{1}{2}at^2})_1}^{x_1} = \overbrace{(\cancel{x_0} + \cancel{v_0}t + \tfrac{1}{2}at^2)_2}^{x_2}$ } Define $x_{10} = x_{20} = 0$

$0 + 27.8\,t_1 + 0 = \frac{1}{2}(2)\,t_2^2$ } where, $t_1 = t_2 = t$

$\Rightarrow t = \frac{27.8}{1} = \underline{27.8\,s}$

Replace $t = 27.8$ in either x_1 or x_2

ie. $\underline{x_1} = 27.8(t_1) = 27.8(27.8) = \underline{773\,m}$

or $\underline{x_2} = \frac{1}{2}(2)(t^2) = (27.8)^2 = \underline{773\,m}$

(b) $\underline{v_c = \text{cnst} = 27.8\,m/s}$

$v_{fpc} = v_{0pc} + a_{pc}t = 0 + (2)(27.8) = \underline{55.6\,m/s} \approx 200\ km/hr$

→ Two objs, one interval, starting at different pts. ($x_{10} \neq x_{20}$) or ($y_{10} \neq y_{20}$)
at different times ($t_1 = t_2 \pm c$)

Ex.
$V_{0A} = +30$ m/s
$h_{building} = 300$ m
$t_A = t_B + 4$
$V_{0B} = 100$ m/s

(a)
$$y_A = y_B$$
final posns when they catch up w/ one another

$$y_0 + v_0 t + \tfrac{1}{2}at^2\Big|_A = y_0 + v_0 t + \tfrac{1}{2}at^2\Big|_B$$

$$300 + 30t_A - 5t_A^2 = 0 + 100t_B - 5t_B^2$$

Define $y_{0B} = 0$ (gr. level)
$y_{0A} = +300$ m.
$a_A = a_B = -10$ m/s²

$$300 + 30(t_B + 4) - 5(t_B + 4)^2 = 100t_B - 5t_B^2$$

$$\cancel{300} + 30t_B + 1\cancel{20} - 5(t_B^2 + 8t_B + 16) = 100t_B - 5t_B^2$$

$$420 + 30t_B - \cancel{5t_B^2} - 40t_B - 80 = 100t_B - \cancel{5t_B^2}$$

$$\Rightarrow 340 - 110t_B = 0 \Rightarrow t_B = \frac{340}{110} = 3.1\text{ s} \Rightarrow t_A = 7.1\text{ s}$$

Final posns at the catch up: $y_A = 300 + 30(7.1) - 5(7.1)^2 = 261$ m

$$y_B = 100(3.1) - 5(3.1)^2 = +262\text{ m}$$
So, $y_A = y_B$ ✓

(b) $v_{fA} = v_{0A} + at_A = 30 - 10(7.1) = -41$ m/s on the way down.

$v_{fB} = v_{0B} + at_B = 100 - 10(3.1) = +69$ m/s " " " up.

A4

→ One obj, Multiple Intervals ($V_{f_1} = V_{0_2}$ at the boundary), with different acc^ns. ie. $a_1 \neq a_2$.

(Ex) $V_0 = 30\ m/s$ = cnst during rx-time ∴ $a_{rx\text{-}time} = 0$

$X_{m_0} = 70\ m$

$t_{rx} = 0.5\ s$

$a_{after\ break} = -8\ m/s^2$

Define: during rx-time = Interval (1)

after rx-time (brakes pushed) = Interval (2)

∴ Interval (1): $V_{0_1} = 30\ m/s$ = cnst $= V_{f_1}$

$a_1 = 0$

$t_1 = 0.5\ s$

ΔX_1 = Dist. travelled during the rx-time $= V_0 t + \frac{1}{2} a t^2 = 30(0.5) + 0 = 15\ m$

Interval (2): $V_{0_2} = V_{f_1} = 30\ m/s$

$a_2 = -8\ m/s^2$

$V_{f_2} = 0$

ΔX_2 = Dist. needed to stop.

$V_f^2 = V_0^2 + 2a\,\Delta X$

$0 = 30^2 - 16\,\Delta X_2$

$\Rightarrow \Delta X_2 = 56.2\ m$ needed

∴ $\Delta X_1 + \underbrace{\Delta X_2}_{needed} = 15 + 56.2 = 71.2\ m > 70\ m$

∴ There is an accident!

∴ Answer is No!!!

(b) If the driver can not steer to one side, to avoid the acc't.

⇒ V_{f_2} at the acc't (when $\Delta X_2 = 70 - 15 = 55\ m$.)

$V_{f_2}^2 = V_{0_2}^2 + 2a_2\,\Delta X_2 = 30^2 - 16(55) = 900 - 880 = 20\ m^2/s^2$

$\Rightarrow V_f = \pm\sqrt{20} = 4.5\ m/s$.