STAT 3502 B (Winter Term 2019)
ASSIGNMENT 1
Due: Wednesday, February 13, 2019, in class.

NOTE: 1. Late assignments will NOT be accepted.
2. TAs will NOT accept assignments directly from students.
3. Total mark = 100. Marks for individual questions are given in [ ].

1. In a lot of 50 light bulbs, there are 2 bad bulbs. An inspector examines 5 bulbs, which are selected at random and without replacement.

[5] (a) Find the probability of at least 1 defective bulb among the 5.

[5] (b) How many bulbs should be examined so that the probability of finding at least 1 bad bulb exceeds 1/2?

2. A town had two fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

[5] (a) What is the probability that neither is available when needed?

[5] (b) What is the probability that a fire engine is available when needed?

3. A fair coin is to be tossed as many times as necessary to turn up one head. Thus the elements s of the sample space S are H, TH, TTH, $TTTH$, and so forth.

[5] (a) Show that $\mathbf{P}(S) = 1$. (**Hint:** Use the formula for an infinite geometric series.)

[5] (b) Let $A_1 = \{H, TH, TTH, TTTH, TTTTH\}$ and $A_2 = \{TTTTH, TTTTTH\}$. Compute $\mathbf{P}(A_1)$, $\mathbf{P}(A_2)$, $\mathbf{P}(A_1 \cap A_2)$, and $\mathbf{P}(A_1 \cup A_2)$.

4. This problem deals with generalization of the Addition Law to the case of three events.

[5] (a) Prove that for every three events A_1, A_2, and A_3,

$$\begin{aligned}\mathbf{P}(A_1 \cup A_2 \cup A_3) &= \mathbf{P}(A_1) + \mathbf{P}(A_2) + \mathbf{P}(A_3) \\ &\quad - \mathbf{P}(A_1 \cap A_2) - \mathbf{P}(A_1 \cap A_3) - \mathbf{P}(A_2 \cap A_3) + \mathbf{P}(A_1 \cap A_2 \cap A_3).\end{aligned}$$

[5] (b) In a certain city, three newspapers A, B, and C are published. Suppose that 60% of the families in the city subscribe to newspaper A, 40% of the families subscribe to newspaper B, and 30% subscribe to newspaper C. Suppose also that 20% of the families subscribe to both A and B, 10% subscribe to both A and C, 20% subscribe to both B and C, and 5% subscribe to all three newspapers A, B, and C. What percentage of the families in the city subscribe to at least one of the three newspapers? (**Hint:** Use part (a).)

5. An electrical system consists of five components as illustrated in Figure 1. The system works if either the components A and B work or the components C, D, and E work. The reliability (probability of working) of each component is also shown in Figure 1. Assume the components fail independently.

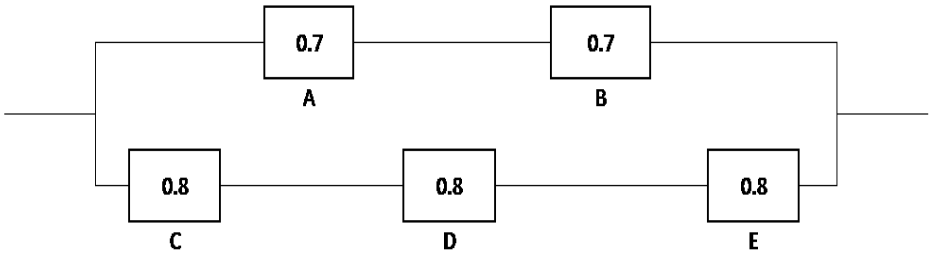

Figure 1: *Diagram for Problem 5.*

[5] (a) What is the probability that the entire system works?

[5] (b) Given that the system works, what is the probability that component A is not working?

6. In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective.

[5] (a) If a finished product is randomly selected, what is the probability that it is defective?

[5] (b) If a product was chosen randomly and found to be defective, what is the probability that it is made by machine B_3?

7. Suppose that A, B, and C are tree independent events such that $\mathbf{P}(A) = 1/4$, $\mathbf{P}(B) = 1/3$, and $\mathbf{P}(C) = 1/2$.

[5] (a) Determine the probability that none of these three events will occur.

[5] (b) Determine the probability that exactly one of these three events will occur.

8. Suppose that a box contains seven red balls and three blue balls. Suppose that five balls are selected at random, without replacement, and let X be the number of red balls that will be obtained.

[5] (a) Determine the probability mass function (pmf) of X.

[5] (b) Determine and sketch the cumulative distribution function (cdf) of X.

[5] (c) Find the expected value of X.

9. An investment firm offers its customers municipal bonds that mature after varying numbers of years. The cdf of X, the number of years to maturity for a randomly selected bond, is given by

$$F(x) = \begin{cases} 0, & x < 1, \\ 1/4, & 1 \le x < 3, \\ 1/2, & 3 \le x < 5, \\ 3/4, & 5 \le x < 7, \\ 1, & x \ge 7. \end{cases}$$

[5] (a) Determine the pmf of X.

[5] (b) Using just the cdf, compute $\mathbf{P}(X = 4)$, $\mathbf{P}(0.4 \leq X < 5)$, and $\mathbf{P}(X < 5 | X \geq 2)$.

[5] 10. A large industrial firm purchases several new word processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of word processors, X, purchased each year has the following probability distribution:

x	0	1	2	3
$p(x)$	1/10	3/10	2/5	1/5

If the cost of the desired model is \$1200 per unit and at the end of the year a refund of $50X^2$ dollars will be issued, how much can this firm expect to spend on new word processors during this year?