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VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

Business Analytics - ADM 2302 M, N, P and Q

Midterm Exam - Winter 2019 SOLUTIONS

Duration: 2 hours

Professors: Jonathan Li and Rim Jaber

Last Name: ________________________ **First Name:** _____________________

Student #: ________________________ **Section:** _____________________

Instructions:

1- Write in your last and first name, your Student ID number, and your section in the spaces above, and sign the Statement of Academic Integrity below.
2- Verify that your exam copy has 8 pages (including this title page). If yours does not, please inform the professor now.
3- Answer all questions on this examination copy. **Use the opposite (blank) side, if necessary**. Only answers in this exam booklet will be marked. Show all work.
4- This is a closed-book exam: however, one double-sided cheat sheet letter (8.5" x 11") paper, and a calculator are allowed for arithmetic use only. NO restrictions on the content of the sheet, however **it must be handed over with the exam copy at the end of the exam period, or else your copy will not be marked.**
5- Read each question very carefully: only provide what is asked and do not hesitate to ask for clarifications when needed.
6- **NO COMMUNICATION DEVICES MAY BE WITHIN SIGHT.**
7- Good luck and keep smiling.

DO NOT WRITE ON THE TABLE BELOW

Questions	1	2	3	4	Total
Notes					
Points	25	27	24	24	100

Statement of Academic Integrity
The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

Statement to be signed by the student:
I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed:____________________________________

Note: an examination copy or booklet without that signed statement will not be graded and will receive a final exam grade of zero.

Question 1: Graphical Method (25 points)

"Innis Investments" manages funds for a number of companies and wealthy clients. The investment strategy is tailored to each client's needs. For a new client, Innis has been authorized to invest up to 1.2 million in two investments funds: a stock fund and a money market fund. Each unit of the stock costs $50 and provides and annual rate return of 10%; each unit of the money market fund costs $100 and provides an annual rate of return of 4%.

The client wants to minimize risk subject to the requirements that the annual income from the investments be at least $60,000. According to the Innis's risk measurement system, each unit invested in stock fund has a risk index of 8, and each unit invested in the money market fund has a risk index of 3; the higher risk index associated with the stock fund simply indicates that it is the riskier investment. Innis's client has also specified that at least $300,000 be invested in the money market fund.

Below is the correct linear programming formulation for the "Innis Investments" problem that will minimize the total risk index for the portfolio:

Let x1 = the number of units of stock fund purchased for the client
x2 = the number of units of money market fund purchased for the client

Minimize $Z = 8x1 + 3x2$

Subject to

$50x1 + 100x2 \leq 1{,}200{,}000$ (Funds available)

$5x1 + 4x2 \geq 60{,}000$ (Annual income)

$x2 \geq 3{,}000$ (Minimum units in money market funds)

$x1, x2 \geq 0$

(a) Using the graphical method, determine the optimal solution: **How many units of each fund "Innis investments"** should purchase for the client and what is the minimum total risk index for the portfolio. (16 points)

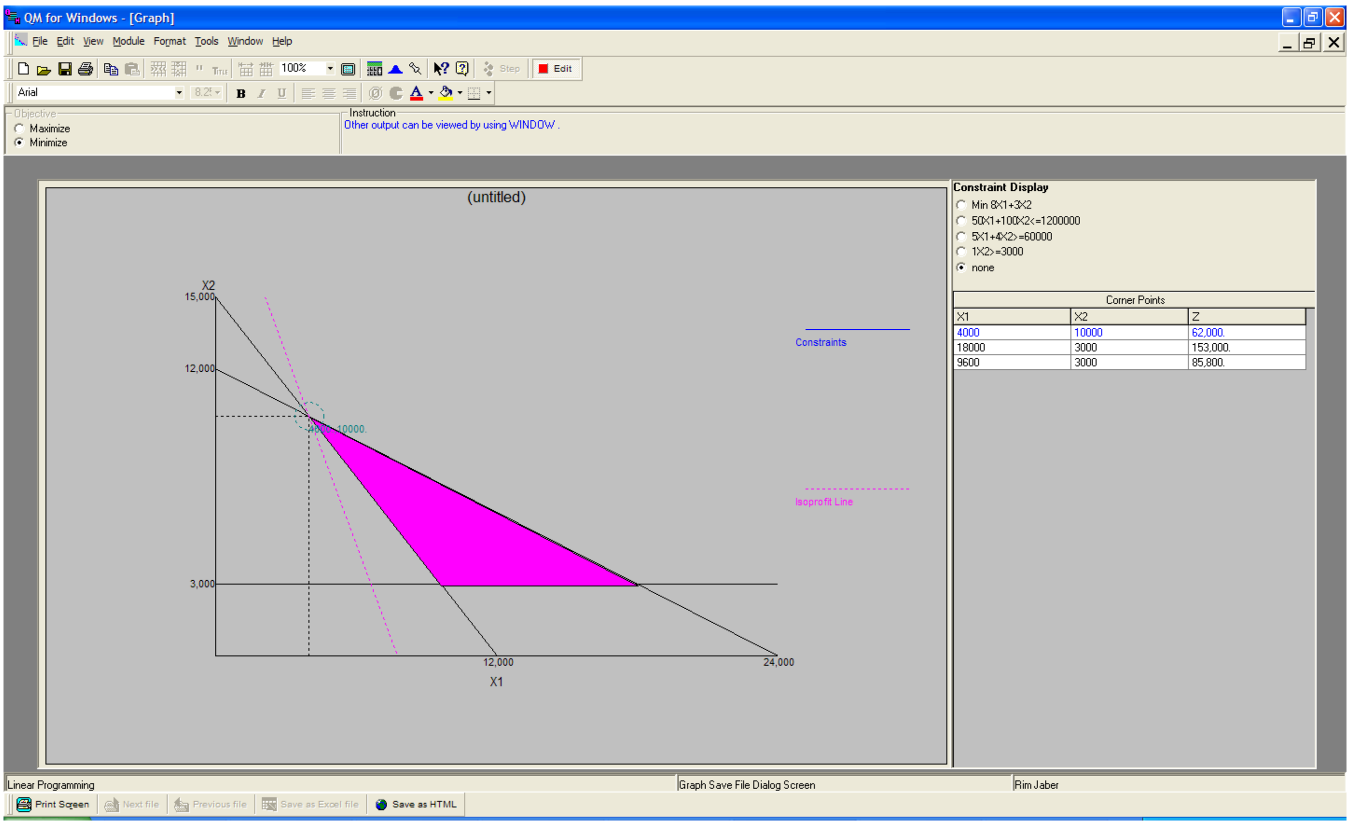
Make sure to graph the constraint lines and mark them clearly with the numbers (1), (2) and (3) to indicate which line corresponds to which constraint and darken the feasible region. Provide all necessary details to justify your answers. (Extra blank space is available on the next page 3).

Marking scheme:

- 6 points for drawing the constraints
- 3 points finding correct feasible region
- 6 points for finding the optimal solution using either the corner point method or the isocost method:

Showing the work using isocost line method or corner point method (3 points)

x1 = 4,000 , x2 = 10,000 (2 points) and value of the objective function = 62,000 (2 points)



(b) How much annual income will this investment strategy generate? Justify. (3 points)

Annual Income = 5(4000) + 4 (10,000) = 60,000

(c) Are any constraints redundant? If so which one? (3 points)

The non-negativity constraints are redundant constraint.

(d) Are any constraints binding? If so which one? (3 points)

Constraint 1 (50x1 + 100x2 <= 1,200,000) and Constraint 2 (5x1 + 4x2 >= 60,000)

Question 2: Linear Programming Formulation (27 points)

Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches. They would like to keep their costs down, but they also need to meet the nutritional requirements of the children. They have already decided to go with the peanut butter and jelly sandwiches, and some combination of milk and orange juice. The nutritional content of each food choice and its cost are given in the table below:

Food item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (cents)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp.)	75	100	0	4	4
Strawberry jelly (1 tbsp.)	0	50	3	0	7
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows:

- Each child should receive between 400 and 600 calories.
- No more than 30% of the total calories should come from fat.
- Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein.
- Furthermore, for practical reasons:
  - each child needs **exactly** 2 slices of bread (to make the sandwich),
  - **at least twice** as much peanut butter as jelly,
  - and **at least** 1 cup of liquid (milk **OR** juice).

Joyce and Marvin would like to select the food choices for each child that minimize cost **(in dollars, $)** while meeting the above requirements.

Formulate the corresponding linear programming model. Define the decision variables, objective function, and constraints. **DO NOT SOLVE.**

Solution

Decision Variables **(3 points)**

B = slices of bread,
P= Tablespoons of peanut butter,
S = Tablespoons of strawberry jelly,

M = cups of milk,
J = cups of juice.

Minimize $C = \$0.05B + \$0.04P + \$0.07S + \$0.15M + \$0.35J$ **(3 points)**

subject to $70B + 100P + 50S + 150M + 100J \geq 400$ calories **(2 points)**
$70B + 100P + 50S + 150M + 100J \leq 600$ calories **(2 points)**

$10B + 75P + 70M \leq 0.3(70B + 100P + 50S + 150M + 100J)$ **(3 points)**

$3S + 2M + 120J \geq 60$mg Vitamin C **(2 points)**
$3B + 4P + 8M + J \geq 12$mg Protein **(2 points)**
$B = 2$ slices **(2 points)**

$P \geq 2S$ ***(3 points)***
$M + J \geq 1$ cup **(3 points)**

and $B \geq 0, P \geq 0, S \geq 0, M \geq 0, J \geq 0.$ **(2 points)**

Question 3: Linear Programming Formulation (24 points)

A manufacturing firm wishes to schedule production for 3 months in advance to meet known monthly demands. The firm can produce 1,500 items per month on regular shifts at a unit cost of $13 per item and by using overtime the firm can produce up to 300 additional items per month at unit cost of $7 per item more than the unit cost for regular production. It costs $3 per month to hold an item in inventory. Monthly sales will be, respectively, 1,100, 1,500, and 2,100. The firm wants to have at least 100 units in inventory at the end month 3.

A labour agreement going into effect on at the start of the third month will raise the unit cost by 10% for both regular and overtime shift.

The firm wishes to determine an optimal production schedule which minimizes total costs of production and storage (i.e. inventory holding). Formulate the above as a linear programming problem. **Define the decision variables, objective function, and constraints. DO NOT SOLVE**

Decision Variables (4 points)

Xi = # of units to produce during regular shift in period j
Yj = # of units to produce during overtime shift in period j
Ij = # of units inventory at the end of period j
Where j = 1, 2,3

Minimize Z = $13X_1+13X_2+$ **$14.3X_3$**$+ 20Y_1+20Y_2+$ **$22Y_3$**$+3I_1+3I_2+3I_3$ **(6 points: 2 points for each regular, overtime and inventory)**

Subject to
Demand Constraints **(6 points: 2 points each)**

$X_1 + Y_1 – I_{A1} = 1,100$ (month 1)
$I_1 + X_2 + Y_2 – I_2 = 1500$ (month 2)
$I_2 + X_3 + Y_3 – I_3 = 2,100$ (month 3)

Production constraints
X_1 =< 1500; X_2 =< 1500; X_3 <= 1500 **(2 points)**

Y_1 <= 300; Y_2 <= 300; Y_3 <= 300 **(2 points)**

Month 3 ending inventory **(2 points)**
I_3 >= 100

Xj, Yj, Ij >= 0 for all j = 1, 2, 3 **(2 points)**

QUESTION 4 (24 points): Linear Programming – Solver Output and Sensitivity

Consider a pharmaceutical company that is looking to create a treatment that requires the patient to consume at least 3 12-ounce servings per day. The treatment consists of four ingredients – A, B, C and D. Each of the four ingredients contains various levels of three chemicals that we will call X, Y and Z. The number of units of each chemical present in each ingredient is given in the table below as are the costs per ounce of each ingredient.

	Ingredient A	Ingredient B	Ingredient C	Ingredient D
Chemical X	3	4	8	10
Chemical Y	5	3	6	6
Chemical Z	10	25	20	40
Cost per ounce	$0.40	$0.20	$0.60	$0.30

Health regulations require that the dosage consumed per day should contain a minimum of 280 units of Chemical X and a minimum of 200 units of chemical Y. Furthermore, the total number of units of chemical Z consumed can be no more than 1050 units.
An enterprising member of the company recalled having taken a course way back in 2nd year of her undergraduate degree at Telfer that would help the company compose the treatment at minimal cost. She consequently formulated this problem as a linear programming model and solved it using Excel. (Her bosses were very impressed and gave her a substantial raise.) Below is the Answer Report that she provided to her bosses. Unfortunately, a jealous co-worker has sabotaged her work as evident by missing values in some cells (indicated by a dark rectangle).

(a) Help her find the missing optimal value for Ingredient A. Full marks only accorded if you show your work (4 points).

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$F$5	Cost	0	13.0625

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Ingredients A	0		Contin
$C$2	Ingredients B	0	0	Contin
$D$2	Ingredients C	0	4.125	Contin
$E$2	Ingredients D	0	21.625	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$11	Chemical Z LHS	1050	$F$11<=$H$11	Binding	0
$F$8	Daily Dose LHS	36	$F$8>=$H$8	Binding	0
$F$9	Chemcial X LHS	280	$F$9>=$H$9	Binding	0
$F$10	Chemical Y LHS	205.75	$F$10>=$H$10	Not Binding	5.75

Objective function is 0.4A+0.2B+0.6C+0.3D = 13.0625

So, A = (13.0625 – 0.2*0 – 0.6*4.125 – 0.3*21.625)/0.4 = 10.25

(b) What would happen to the optimal solution and the optimal objective value if the limit on the Chemical Y was increased to 204? (3 points)

Nothing as it is a non-binding constraint and the amount of chemical Y currently used in the optimal solution is greater than 204.

(c) As a follow-up her bosses asked her a number of questions which she answered using the sensitivity report given below. Provide your answers to the questions along with justification.

i) What is the impact on the optimal value of objective function if the treatment must contain 1 ounce of ingredient B (round to the nearest cent)? (3 points)

Reduced cost indicates that each ounce of ingredient B will cause the total cost per daily dosage to increase by $0.069 . The new cost will therefore be $13.06 + 0.069 = $13.13.

ii) There is some concern that the cost of an ounce of ingredient C may jump **to** $1.00 (its current cost is $0.6). What is the impact on the optimal solution as well as the optimal objective function value? Make sure to justify your answer. (4 points).

Justification must include the fact that the increase to $1.00 is below the allowable increase with upper bound of $2.10. (1 point). Thus the optimal solutions remain the same (1 point), So new cost is $13.06 + 0.4*4.125 = 14.71. (2 points)

iii) What is the impact on the optimal value of the objective function if units of chemical Z consumed can be no more than 1000 units (round to the nearest cent)? (3 points)

The decrease to 1000 (a decrease of 50 units) is still in the allowable range (allowable decrease of 346). Thus the shadow price value remains valid. (1 point).
P.S.: Optimal solution will change because the it is a binding constraint. **deduct 1 point if student say optimal solution will NOT change)**
The impact on the optimal cost would be: $13.06+ (-50)*(- 0.02375)= $13.06+ $1.1875 = $14.2475 = $14.25 (2 points)

iv) Previously, an enterprising former Telfer student had solved the same problem but for a daily dosage of 40 ounces. She had determined an optimal cost of 14.56. Use this to determine the impact on the optimal cost for increasing the daily dosage to 45. (5 points)

Shadow price can be calculated as (14.56-13.06)/(40-36) = 0.375. (2 points)
The increase to 45 (45 – 36 = 9) is still in the allowable range (allowable decrease of 16.5) thus the shadow price does NOT change! (1 point).
P.S.: Optimal solution will change because the it is a binding constraint. (**deduct 1 point if student say optimal solution will NOT change)**
The impact on the optimal cost would be 13.06+0.375*(45-36) = 16.435 (2 points)

v) Could we do the same analysis a part iv) for an increase in the daily dosage to 55 ounces? (2 points)

No, because this would land it outside the allowable range.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Ingredients A	10.25	0	0.4	0.061111111	0.25
$C$2	Ingredients B	0	0.06875	0.2	1E+30	0.06875
$D$2	Ingredients C	4.125	0	0.6	1.5	0.073333333
$E$2	Ingredients D	21.625	0	0.3	0.084615385	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Chemical Z LHS	1050	-0.02375	1050	47.14285714	346
$F$8	Daily Dose LHS	36	[redacted]	36	16.5	1.277777778
$F$9	Chemcial X LHS	280	0.0875	280	41	11
$F$10	Chemical Y LHS	205.75	0	200	5.75	1E+30