

ELG6372 Principles of Photonics

Ch. 1 Electromagnetic Fields and Waves

1.1 Introduction

EM field described by $\mathcal{E} \rightarrow$ electric field vector

$\mathcal{H} \rightarrow$ magnetic field vector

Effect of EM field on media is quantified by constitutive relations

$$\text{electric displacement } \mathcal{D} = \epsilon \mathcal{E} = \epsilon_0 \mathcal{E} + \mathcal{P} \quad (1.1)$$

\swarrow permittivity \searrow electric polarization

$$\text{magnetic induction } \mathcal{B} = \mu \mathcal{H} = \mu_0 (\mathcal{H} + \mathcal{M}) \quad (1.2)$$

\swarrow permeability \searrow magnetic polarization

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$\epsilon_0, \mu_0 \rightarrow$ "free space" quantities

Maxwell's equations

$$\text{Faraday's law } \nabla \times \mathcal{E} + \frac{\partial \mathcal{B}}{\partial t} = 0 \quad (1.3)$$

$$\text{Ampere's law } \nabla \times \mathcal{H} - \frac{\partial \mathcal{D}}{\partial t} = \mathcal{J} \quad (1.4)$$

$$\text{Gauss' law – electric } \nabla \cdot \mathcal{D} = \rho \quad (1.5)$$

$$\text{Gauss's law – magnetic } \nabla \cdot \mathcal{B} = 0 \quad (1.6)$$

\uparrow electric current density (A/m²)

\uparrow electric charge density (C/m²)

Boundary Conditions

What are the continuity relations between field components across an interface?

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(a) Normal components

Recall Gauss divergence theorem:

$$\int_V \nabla \cdot \mathbf{F} dV = \oint_S \mathbf{F} \cdot d\mathbf{S} \quad (1.7)$$

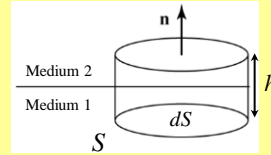


Figure 1.1

(1.7) applied to (1.5) & (1.6) yields, as $h \rightarrow 0$,

$$\hat{\mathbf{n}} \cdot (\mathcal{B}_2 - \mathcal{B}_1) = 0 \quad (1.8)$$

$$\hat{\mathbf{n}} \cdot (\mathcal{D}_2 - \mathcal{D}_1) = \sigma \quad (1.9)$$

surface charge density, $\sigma dS = \int_V \rho dV$

(b) Tangential components

Recall Stokes theorem:
$$\int_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{l} \quad (1.10)$$

(1.10) applied to (1.3) & (1.4) yields, as $h \rightarrow 0$,

$$\hat{\mathbf{n}} \times (\mathcal{E}_2 - \mathcal{E}_1) = 0 \quad (1.11)$$

$$\hat{\mathbf{n}} \times (\mathcal{H}_2 - \mathcal{H}_1) = \boldsymbol{\kappa} \quad (1.12)$$

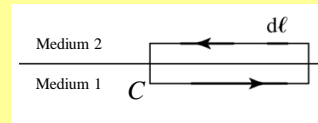


Figure 1.2

surface current density, $\boldsymbol{\kappa} \cdot \hat{\mathbf{b}} dL = \int_S \mathbf{J} \cdot \hat{\mathbf{b}} dS$

Frequently, σ and $\boldsymbol{\kappa}$ are nil, hence across an interface

- tangential components of \mathcal{E} and \mathcal{H} are continuous
- normal components of \mathcal{B} and \mathcal{D} are continuous

Conservation of Energy and Momentum

For a single charge q , moving with velocity \mathbf{v} , the rate at which work is done by the EM field is

$$\frac{dW}{dt} = q \mathbf{v} \cdot \mathcal{E} \quad (1.13a)$$

NB. \mathcal{H} does no work on q because $\mathbf{F}_H \perp \mathbf{v}$

For a continuous distribution of charge and current,

$$\frac{dW}{dt} = \int_V \mathbf{J} \cdot \mathcal{E} dV \quad (1.13b)$$

↑ conversion of EM energy into mechanical/thermal energy

→ energy of the field in V must decrease

(1.4) into (1.13b) yields

$$\int_V \mathbf{J} \cdot \mathcal{E} dV = \int_V \left\{ \mathcal{E} \cdot (\nabla \times \mathcal{H}) - \mathcal{E} \cdot \frac{\partial \mathcal{D}}{\partial t} \right\} dV \quad (1.14)$$

Using the vector identity

$$\nabla \cdot (\mathcal{E} \times \mathcal{H}) = \mathcal{H} \cdot (\nabla \times \mathcal{E}) - \mathcal{E} \cdot (\nabla \times \mathcal{H}) \quad (1.15)$$

and (1.3), (1.14) may be written as

$$\int_V \mathbf{J} \cdot \mathcal{E} dV = - \int_V \left\{ \nabla \cdot (\mathcal{E} \times \mathcal{H}) + \mathcal{E} \cdot \frac{\partial \mathcal{D}}{\partial t} + \mathcal{H} \cdot \frac{\partial \mathcal{B}}{\partial t} \right\} dV \quad (1.16)$$

Let us assume a lossless linear medium without dispersion

→ μ and ϵ are without time dependence

We define the EM field's energy density as

$$U = U_e + U_m = \frac{1}{2} (\mathcal{E} \cdot \mathcal{D} + \mathcal{B} \cdot \mathcal{H}) \quad (1.17)$$

Using (1.17), (1.16) becomes

$$- \int_V \mathbf{J} \cdot \mathcal{E} dV = \int_V \left\{ \frac{\partial U}{\partial t} + \nabla \cdot (\mathcal{E} \times \mathcal{H}) \right\} dV \quad (1.18)$$

We now define the Poynting vector \mathcal{S} (W/m²) as

$$\mathbf{S} = \mathbf{E} \times \mathcal{H} \quad (1.19)$$

Since V is arbitrary, then (1.19) into (1.18) yields the continuity equation, or Poynting's theorem, as

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E} \quad (1.20)$$

Conservation of energy \mathcal{E} may be expressed directly from (1.18), using (1.13b) and (1.20), as

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial t} &= \frac{\partial}{\partial t} (\mathcal{E}_{mech} + \mathcal{E}_{field}) = \frac{\partial W}{\partial t} + \frac{\partial}{\partial t} \int_V U dV \\ &= \int_V \left\{ \mathbf{J} \cdot \mathbf{E} + \frac{\partial U}{\partial t} \right\} dV = - \int_V \nabla \cdot \mathbf{S} dV \end{aligned} \quad (1.21)$$

Applying (1.7) to (1.21) yields

$$\frac{\partial \mathcal{E}}{\partial t} = - \oint_S \hat{\mathbf{n}} \cdot \mathbf{S} dS \quad (1.22)$$

\Rightarrow the flow of \mathbf{S} out of V through S decreases the total energy \mathcal{E}

Now, what about the momentum?

The force on a charged particle may be given as

$$\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (1.23)$$

Integrating over (1.23), using Newton's 2nd law, we can express the net force on all particles in the volume V by the time change in the total particle momentum

$$\frac{\partial \mathcal{P}_{mech}}{\partial t} = \int_V (\rho \mathbf{E} + \mathbf{J} \times \mathbf{B}) dV \quad (1.24)$$

(1.4) and (1.5) into the integrand of (1.24) yields

$$\begin{aligned}\rho\mathbf{E} + \mathbf{J} \times \mathbf{B} &= \mathbf{E}(\nabla \cdot \mathbf{D}) - \mathbf{B} \times \left\{ \nabla \times \mathcal{H} - \frac{\partial \mathbf{D}}{\partial t} \right\} \\ &= \mathbf{E}(\nabla \cdot \mathbf{D}) + \mathbf{B} \times \frac{\partial \mathbf{D}}{\partial t} - \mathbf{B} \times (\nabla \times \mathcal{H})\end{aligned}\quad (1.25)$$

$$\text{Now} \quad \frac{\partial}{\partial t}(\mathbf{D} \times \mathbf{B}) = \frac{\partial \mathbf{D}}{\partial t} \times \mathbf{B} + \mathbf{D} \times \frac{\partial \mathbf{B}}{\partial t}\quad (1.26)$$

Using (1.26), (1.25) becomes

$$\rho\mathbf{E} + \mathbf{J} \times \mathbf{B} = \mathbf{E}(\nabla \cdot \mathbf{D}) - \frac{\partial}{\partial t}(\mathbf{D} \times \mathbf{B}) + \mathbf{D} \times \frac{\partial \mathbf{B}}{\partial t} - \mathbf{B} \times (\nabla \times \mathcal{H})\quad (1.27)$$

Using (1.3) and (1.6), (1.27) becomes

$$\begin{aligned}\rho\mathbf{E} + \mathbf{J} \times \mathbf{B} &= \left[\mathbf{E}(\nabla \cdot \mathbf{D}) - \mathbf{D} \times (\nabla \times \mathbf{E}) + \mathcal{H}(\nabla \cdot \mathbf{B}) - \mathbf{B} \times (\nabla \times \mathcal{H}) \right] \\ &\quad - \frac{\partial}{\partial t}(\mathbf{D} \times \mathbf{B})\end{aligned}\quad (1.28)$$

If we identify the total momentum of the EM field in V as

$$\mathbf{P}_{\text{field}} = \int_V \mathbf{D} \times \mathbf{B} dV\quad (1.29)$$

where the integrand is known as the

$$\text{Minkowski momentum density} \quad g_M = \mathbf{D} \times \mathbf{B}\quad (1.30)$$

then, from (1.24), using (1.28) with (1.29), we have

$$\begin{aligned}\frac{\partial \mathbf{P}}{\partial t} &= \frac{\partial}{\partial t}(\mathbf{P}_{\text{mech}} + \mathbf{P}_{\text{field}}) \\ &= \int_V \left[\mathbf{E}(\nabla \cdot \mathbf{D}) - \mathbf{D} \times (\nabla \times \mathbf{E}) + \mathcal{H}(\nabla \cdot \mathbf{B}) - \mathbf{B} \times (\nabla \times \mathcal{H}) \right] dV\end{aligned}\quad (1.31)$$

Assuming homogeneity, (1.31) becomes

$$\begin{aligned}\frac{\partial \mathbf{P}}{\partial t} &= \int_V \left\{ \varepsilon \left[\mathbf{E}(\nabla \cdot \mathbf{E}) - \mathbf{E} \times (\nabla \times \mathbf{E}) \right] \right. \\ &\quad \left. + \mu \left[\mathcal{H}(\nabla \cdot \mathcal{H}) - \mathcal{H} \times (\nabla \times \mathcal{H}) \right] \right\} dV\end{aligned}\quad (1.32)$$

Recall the vector identity

$$\nabla(\mathcal{A} \cdot \mathcal{B}) = \mathcal{A} \times (\nabla \times \mathcal{B}) + \mathcal{B} \times (\nabla \times \mathcal{A}) + (\mathcal{A} \cdot \nabla) \mathcal{B} + (\mathcal{B} \cdot \nabla) \mathcal{A} \quad (1.33a)$$

This implies

$$\frac{1}{2} \nabla(\mathcal{E} \cdot \mathcal{E}) = \mathcal{E} \times (\nabla \times \mathcal{E}) + (\mathcal{E} \cdot \nabla) \mathcal{E} \quad (1.33b)$$

Consider now the electric field portion of the integrand in (1.32).

Using (1.33b), we find

$$(\mathcal{E} \cdot \nabla) \mathcal{E} - \mathcal{E} \times (\nabla \times \mathcal{E}) = \mathcal{E}(\nabla \cdot \mathcal{E}) + (\mathcal{E} \cdot \nabla) \mathcal{E} - \frac{1}{2} \nabla(\mathcal{E} \cdot \mathcal{E}) \quad (1.34)$$

Inspecting RHS(1.34) shows that its i^{th} component is

$$\text{RHS(1.34)}_i = \sum_j \frac{\partial}{\partial x_j} \left[(\mathcal{E}_i \mathcal{E}_j) - \frac{1}{2} (\mathcal{E} \cdot \mathcal{E}) \delta_{ij} \right] \quad (1.35)$$

There is an expression similar to (1.35) for the magnetic part of the integrand in (1.32). The complete integrand may then be expressed through the *Maxwell stress tensor components*

$$\mathcal{T}_{ij} = \mathcal{E}_i \mathcal{D}_j + \mathcal{H}_i \mathcal{B}_j - \frac{1}{2} (\mathcal{E} \cdot \mathcal{D} + \mathcal{H} \cdot \mathcal{B}) \delta_{ij} \quad (1.36)$$

and the components of (1.32) may then be written as

$$\frac{\partial \mathcal{P}_i}{\partial t} = \sum_j \int_V \frac{\partial \mathcal{T}_{ij}}{\partial x_j} dV \quad (1.37)$$

Using the divergence theorem, this may be written as

$$\frac{\partial \mathcal{P}_i}{\partial t} = \oint_S \sum_j \mathcal{T}_{ij} n_j dS \quad (1.38)$$

The conservation law for momentum is thus

$$\frac{\partial \mathcal{P}}{\partial t} = \oint_S \mathcal{T} \cdot \hat{\mathbf{n}} dS \quad (1.39)$$

NB. $\sum_j \mathcal{T}_{ij} n_j$ is the i^{th} component of the rate at which the momentum per unit area, flowing across S into V , is changing;

i.e., (1.39) gives the force per unit area transmitted across S and acting on the contents of V

Electromagnetic Wave Equations – Time Dependent

For source-free homogeneous, isotropic media...

Rewriting (1.3) & (1.4), using (1.1) & (1.2):

$$\nabla \times \mathbf{E} + \mu \frac{\partial \mathcal{H}}{\partial t} = 0 \quad (1.40)$$

$$\nabla \times \mathcal{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} = 0 \quad (1.41)$$

Taking the curl of (1.40) and interchanging space and time derivatives yields

$$\nabla \times (\nabla \times \mathbf{E}) + \mu \frac{\partial}{\partial t} (\nabla \times \mathcal{H}) = 0 \quad (1.42)$$

(1.41) \rightarrow (1.42) yields

$$\nabla \times (\nabla \times \mathbf{E}) + \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (1.43)$$

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Using a vector identity, (1.43) becomes

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} + \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (1.44)$$

For a source-free region, $\nabla \cdot \mathbf{E}$ vanishes and (1.44) reduces to the wave equation

$$\nabla^2 \mathbf{E} - \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (1.45)$$

Similarly,

$$\nabla^2 \mathcal{H} - \mu \varepsilon \frac{\partial^2 \mathcal{H}}{\partial t^2} = 0 \quad (1.46)$$

(1.45) & (1.46) have plane wave solutions of the form

$$\Psi(\mathbf{r}, t) = \mathbf{A} e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (1.47)$$

with the relation between k and ω given by

$$k^2 - \mu \varepsilon \omega^2 = 0 \quad (1.48)$$

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(1.47) produces sinusoidal field variations in time and space, with identical magnitudes for all \mathbf{r} and t that satisfy

$$\omega t - \mathbf{k} \cdot \mathbf{r} = \text{constant} \quad (1.49)$$

At any instant in time, (1.49) defines a plane normal to \mathbf{k}

→ surface of constant phase

These planar surfaces travel in the direction of \mathbf{k} .

The stationary point of (1.49), $\frac{\partial}{\partial t}(\omega t - \mathbf{k} \cdot \mathbf{r}) = 0$ defines the *phase velocity*

$$\mathbf{v}_p = \hat{\mathbf{k}} \frac{\omega}{k} \quad (1.50)$$

Combining (1.48) and (1.50) permits v_p to be expressed in terms of the material properties of the medium as

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \quad (1.51)$$

More succinctly, $v_p = c/n$ (1.52)

where the *vacuum speed of light* is $c = 1/\sqrt{\mu_0\epsilon_0}$ (1.53)

and the *index of refraction* is

$$n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\mu_r\epsilon_r} \quad (1.54)$$

with ϵ_r the *relative permittivity* and μ_r the *relative permeability*.

Now, consider \mathcal{E} and \mathcal{H} as the vector plane waves

$$\mathcal{E} = \hat{\mathbf{a}}_e E_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (1.55)$$

$$\mathcal{H} = \hat{\mathbf{a}}_h H_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (1.56)$$

Applying (1.5) & (1.6) to (1.55) & (1.56), respectively, we find

$$\mathbf{k} \cdot \hat{\mathbf{a}}_e = \mathbf{k} \cdot \hat{\mathbf{a}}_h = 0 \quad (1.57)$$

so the EM field is a transverse wave.

Using (1.55) & (1.56) in (1.3)...Faraday's law...with (1.1), gives


$$-i\mathbf{k} \times \hat{\mathbf{a}}_e E_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} + i\mu\omega \hat{\mathbf{a}}_h H_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} = 0$$

hence

$$\left(\hat{\mathbf{k}} \times \hat{\mathbf{a}}_e\right) \left(\frac{k}{\mu\omega}\right) E_0 = \hat{\mathbf{a}}_h H_0 \quad (1.58)$$

Field directions, from (1.58), are constrained as

$$\hat{\mathbf{a}}_h = \hat{\mathbf{k}} \times \hat{\mathbf{a}}_e \quad (1.59)$$



a mutually orthogonal triplet

and the field magnitudes, using (1.48), as

$$H_0 = \left(\frac{k}{\mu\omega}\right) E_0 = \sqrt{\frac{\varepsilon}{\mu}} E_0 \equiv \eta^{-1} E_0 \quad (1.60)$$

where the *intrinsic impedance* of the medium is $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ (1.61)

Time Averaged Properties

→ optical detectors respond much slower than the optical period

NB. The time-harmonic monochromatic field is the real part of the complex form

→ terms quadratic in the field need to be treated carefully

Consider the functions

$$A(t) = A_0 \cos(\omega t + \alpha) \quad (1.62)$$

$$B(t) = B_0 \cos(\omega t + \beta) \quad (1.63)$$

The time average is

$$\begin{aligned} \langle A(t)B(t) \rangle &= \int_0^T A_0 B_0 \cos(\omega t + \alpha) \cos(\omega t + \beta) dt \\ &= \frac{A_0 B_0}{2T} \int_0^T \{ \cos(2\omega t + \alpha + \beta) + \cos(\alpha - \beta) \} dt \end{aligned}$$

Since the first term is periodic in T (NB. $\omega = 2\pi/T$), and the second term is a constant, the time average is

$$\langle A(t)B(t) \rangle = \frac{1}{2} A_0 B_0 \cos(\alpha - \beta) \quad (1.64)$$

In the complex formalism, (1.62) and (1.63) are

$$A(t) = A_0 e^{i(\omega t + \alpha)} \equiv A_s e^{i\omega t} \quad (1.65)$$

$$B(t) = B_0 e^{i(\omega t + \beta)} \equiv B_s e^{i\omega t} \quad (1.66)$$

Thus, (1.64) may be written as

$$\langle A(t)B(t) \rangle = \frac{1}{2} A_0 B_0 \operatorname{Re} \left\{ e^{i(\alpha - \beta)} \right\} = \frac{1}{2} \operatorname{Re} \left\{ A_s B_s^* \right\} \quad (1.67a)$$

Hence, the time average of quantities quadratic in the field, in the complex formalism, is

$$\langle \operatorname{Re} [A(t)] \operatorname{Re} [B(t)] \rangle = \frac{1}{2} \operatorname{Re} \left\{ A(t) B^*(t) \right\} \quad (1.67b)$$

The time-averaged Poynting vector, from (1.19) and (1.67b), is then

$$\langle \mathbf{S} \rangle = \frac{1}{2} \operatorname{Re} \left\{ \mathbf{E} \times \mathcal{H}^* \right\} \quad (1.68)$$

For plane waves (1.55) & (1.56), using (1.60), (1.68) becomes

$$\begin{aligned} \langle \mathbf{S} \rangle &= \frac{1}{2} \operatorname{Re} \left\{ \left[\hat{\mathbf{a}}_e E_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \right] \times \left[\hat{\mathbf{a}}_h H_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \right]^* \right\} \\ &= \frac{1}{2} \operatorname{Re} \left\{ (\hat{\mathbf{a}}_e \times \hat{\mathbf{a}}_h) E_0 H_0 \right\} = \frac{1}{2} \hat{\mathbf{k}} E_0 H_0 = \frac{\hat{\mathbf{k}} E_0^2}{2\eta} \end{aligned} \quad (1.69)$$

NB. Energy flux defines the direction of propagation, here parallel to the wavevector.

The time-averaged energy density, from (1.17) and (1.67b), is

$$\langle U \rangle = \frac{1}{4} \operatorname{Re} \left\{ \mathbf{E} \cdot \mathcal{D}^* + \mathcal{B} \cdot \mathcal{H}^* \right\} \quad (1.70)$$

For plane waves (1.55) & (1.56), using (1.1), (1.2) and (1.61), (1.70) becomes

$$\begin{aligned}\langle U \rangle &= \frac{1}{4} \operatorname{Re} \left\{ \mathbf{E} \cdot (\varepsilon \mathbf{E})^* + (\mu \mathbf{H}) \cdot \mathbf{H}^* \right\} \\ &= \frac{1}{4} \left\{ \varepsilon E_0^2 + \mu H_0^2 \right\} = \frac{1}{4} \left\{ \varepsilon E_0^2 + \mu \eta^{-1} E_0^2 \right\} = \frac{1}{2} \varepsilon E_0^2 \quad (1.71)\end{aligned}$$

Comparing (1.69) and (1.71) reveals that the energy velocity

$$v_e = \frac{\langle S \rangle}{\langle U \rangle} = \frac{E_0^2 / 2\eta}{\frac{1}{2} \varepsilon E_0^2} = \frac{1}{\varepsilon \eta} = \frac{1}{\sqrt{\mu \varepsilon}} = v_p \quad (1.72)$$

and thus, energy flows at the phase velocity.

NB. This is only true for monochromatic plane waves in a lossless, dispersionless, isotropic medium.

Electromagnetic Wave Equations – Time Independent

For monochromatic waves (not necessarily plane waves), we can employ the complex representation to remove the time dependence

– cf. (1.55) & (1.56)

$$\mathbf{E}(\mathbf{r}, t) \rightarrow \operatorname{Re} \left\{ \mathbf{E}(\mathbf{r}) e^{i\omega t} \right\} \quad (1.73)$$

$$\mathbf{H}(\mathbf{r}, t) \rightarrow \operatorname{Re} \left\{ \mathbf{H}(\mathbf{r}) e^{i\omega t} \right\} \quad (1.74)$$

...similarly for $\mathcal{P} \rightarrow \mathbf{P}$, $\mathcal{D} \rightarrow \mathbf{D}$, $\mathcal{B} \rightarrow \mathbf{B}$, $\mathcal{M} \rightarrow \mathbf{M}$

Thus, in the frequency domain, the constitutive equations (1.1) & (1.2) become

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad (1.75)$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad (1.76)$$

Defining time independent electric charge and electric current densities

$$\rho(\mathbf{r}) = e^{-i\omega t} \rho(\mathbf{r}, t) \quad (1.77)$$

$$\mathbf{J}(\mathbf{r}) = e^{-i\omega t} \mathbf{J}(\mathbf{r}, t) \quad (1.78)$$

the Maxwell's equations (1.3) – (1.6) become

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B} \quad (1.79)$$

$$\nabla \times \mathbf{H} = i\omega \mathbf{D} + \mathbf{J} \quad (1.80)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (1.81)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1.82)$$