MAT 2377 B: Probability and Statistics for Engineers
Midterm Examination

Duration: 80 minutes **Tuesday, February 26, 2019**

Professor: Rachid Bentoumi.

Name: ____________________ Student Number: ____________________

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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- This is a closed book examination. One single-sided sheet is permitted.
- Only Faculty standard calculators are permitted.
- There are 12 multiple choice questions in total. Record your answers in the table below.

Question	Answer	Question	Answer	Question	Answer
1	D	5	A	9	A
2	D	6	A	10	E
3	E	7	E	11	D
4	C	8	E	12	D

Note: at the end of the examination, hand in only this page.

Multiple Choice Questions (1 mark each)

1. Let X be a discrete random variable with the following probability mass function $f_X(x)$.

x	0	1	2	3	4
$f_X(x)$	0.05	0.25	0.33	0.09	0.28

Calculate $P(X - E[X] < 0.5)$.

A) 0.77 B) 0.16 C) 0.45 D) 0.63 E) 0.19

answer = D

Solution: First, we compute the expected value of the discrete random variable X

$$E[X] = 0(0.05) + 1(0.25) + 2(0.33) + 3(0.09) + 4(0.28) = 2.3.$$

It follows that,

$$\begin{aligned}
P(X - E[X] < 0.5) &= P(X < E[X] + 0.5) \\
&= P(X < 2.3 + 0.5) \\
&= P(X < 2.8) \\
&= P(X \leq 2) \\
&= P(X = 0) + P(X = 1) + P(X = 2) \\
&= 0.05 + 0.25 + 0.33 \\
&= 0.63
\end{aligned}$$

2. Any accident involving a plane needs to be thoroughly investigated. If it results from a structural defect, the probability of it being recognized as such is 0.9. If cause of the accident is not a structural defect, the probability that it is wrongly attributed to a structural failure is 0.2. Suppose that 25% of all accidents occur as a result of structural defect. Considering an accident, what is the probability that the cause is attributed to a structural defect?

A) 0.6250 B) 0.4555 C) 0.3245 D) 0.3750 E) 0.2500

answer = D

Solution: Let D be the event that the result of the accident is a structural defect and let A be the event that the cause of the accident is attributed to a structural defect. We have $P(D) = 0.2$, $P(A|D) = 0.9$, and $P(A|D') = 0.2$. By the total probability rule, the probability that the cause of the accident is attributed to a structural defect is

$$\begin{aligned} P(A) &= P(A|D)P(D) + P(A|D')P(D') \\ &= (0.9)(0.25) + (0.2)(0.75) \\ &= (0.9)(0.25) + (0.2)(0.75) \\ &= 0.375 \end{aligned}$$

3. Let X be a discrete random variable with support $\{0, 1, 2, 3, 4\}$. Its cumulative distribution function $F_X(x)$ is given as follows.

x	0	1	2	3	4
$F_X(x)$	0.2	0.3	0.4	0.8	1

Compute $E[X]$.

A) 2.7 B) 3.0 C) 7.5 D) 2.0 E) 2.3

answer = E

Solution: The probability mass function is

$$f(x) = \begin{cases} 0.2; & \text{if } x = 0 \\ 0.3 - 0.2 = 0.1; & \text{if } x = 1 \\ 0.4 - 0.3 = 0.1; & \text{if } x = 2 \\ 0.8 - 0.4 = 0.4; & \text{if } x = 3 \\ 1.0 - 0.8 = 0.2; & \text{if } x = 2 \end{cases}$$

Thus, the expected value of X is

$$E[X] = \sum_{x \in R_X} x\, f(x) = 0(0.2) + 1(0.1) + 2(0.1) + 3(0.4) + 4(0.2) = 2.3.$$

4. Suppose there are 14 boys and 6 girls in a classroom. A teacher randomly selects 5 students from the class without replacement. What is the probably that the selected students contain 2 girls?

A) 0.079 B) 0.001 C) 0.352 D) 0.309 E) 0.024

answer is C.

Solution:

Let A be the event that the selected students contain 2 girls (and 3 boys). Note that order of selection is not important, we have

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}_6C_2 \times {}_{14}C_3}{{}_{20}C_5} = 0.352$$

5. If an electrical switch is kept in dry environment, it fails to work with probability 1%. If the switch is kept in humid environment, the failure probability is 8%. Assume that 90% of switches are kept in dry environment, whereas the remaining 10% are kept in humid environment. If a switch fails to work, what is the probability that it is kept in humid environment?

A) 0.4706 B) 0.6783 C) 0.0028 D) 0.9991 E) 0.5567

answer = A

Solution: Let F-"failure", H-"humid", D-"dry".

Given: $P(F|D) = 0.01$, $P(F|H) = 0.08$, $P(D) = 0.9$, $P(H) = 0.1$, we have

$$P(F) = P(F|D)P(D) + P(F|H)P(H) = (0.01)(0.9) + (0.08)(0.1) = 0.009 + 0.008 = 0.017$$

We want to find

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F|H)P(H)}{P(F)} = \frac{(0.08)(0.1)}{0.017} = 0.4706$$

6. An electronic system consists of 30 independent integrated circuits. Each of these circuits has a probability of 0.0175 of being faulty. The system

is operational when there is at most one faulty circuit. What is the probability that this system is operational?

A) 0.9034 B) 0.5888 C) 0.3146 D) 0.9825 E) 0.9245

answer = A

Solution: Let X be the number of faulty circuits among the 30 circuits. We have $X \sim B(n = 30; p = 0.0175)$. The probability that the system is operational is

$$P(X \leq 1) = \binom{30}{0} p^0 (1-p)^{30} + \binom{30}{1} p^1 (1-p)^{29} = 0.9034,$$

where $p = 0.0175$.

7. Earthquakes in a given region follow a Poisson process with rate 0.05 per month. What is the probability that we will observe at most 1 earthquake in this region within 12 months?

A) 0.9012 B) 0.1219 C) 0.3292 D) 0.0988 E) 0.8781

answer = E

Solution: Let X be number of earthquakes observed within 12 months. Then, we have $X \sim Poisson(0.05 \times 12) = Poisson(0.6)$. Thus,

$$P(X \leq 1) = f_X(0) + f_X(1) = e^{-0.6}\frac{0.6^0}{0!} + e^{-0.6}\frac{0.6^1}{1!} = 0.8781.$$

8. Many students at uOttawa are bilingual, that is they are fluent in both French and English. However, some students are only fluent in English and some are only fluent in French. Furthermore, all students at uOttawa are fluent in either French or English.

Suppose that we randomly select a uOttawa student. Let A be the event that the student is **only** fluent in English and let B be the event that the student is **only** fluent in French.

Which of the following statements is **incorrect**?

A) $P(A|B) = P(B|A)$
B) $P(A \cup B) = P(A) + P(B)$

C) $P(A^c \cap B) = P(B)$
D) A and B are mutually exclusive.
E) A and B are independent.

answer = E

Solution: Note that A and B are mutually exclusive by definition. Thus, $P(A \cap B) = 0$. This immediately implies that they are not independent, because the occurrence of A implies the failure of B. Mathematically,

$$P(A \cap B) = 0 \neq P(A)P(B) > 0.$$

Thus, A and B are dependent. The answer is E.

Using the fact $P(A \cap B) = 0$, we can easily verify $P(A|B) = P(B|A) = 0$, $A^c \cap B = B$, and $P(A \cup B) = P(A) + P(B)$ in this case.

9. Let X, Y be two discrete random variables, whose joint probability mass function is given as follows.

x	y	$f_{XY}(x, y)$
1	1	0.1
1	2	0.3
1	4	0.2
2	1	0.1
2	2	0.2
2	4	0.1

Compute $P(3 < X + Y \leq 5)$.

A) 0.4 B) 0.3 C) 0.1 D) 0 E) 0.6

answer = A

Solution: We want

$$\begin{aligned} P(3 < X + Y \leq 5) &= \sum_{(x,y) \in R_{XY} \; : \; 3 < x+y \leq 5} f_{XY}(x, y) \quad \text{(by additivity)} \\ &= f_{XY}(1, 4) + f_{XY}(2, 2) \\ &= 0.2 + 0.2 = 0.4. \end{aligned}$$

10. Let X be a discrete random variable with mean $\mu_X = 12.5$ and variance $\sigma_X^2 = 0.36$. Let $Y = -2X + 30$. Find the mean and the standard deviation of Y.

A) $\mu_Y = 6, \quad \sigma_Y = -1.2$ B) $\mu_Y = 5, \quad \sigma_Y = -0.72$
C) $\mu_Y = 5, \quad \sigma_Y = 0.72$ D) $\mu_Y = 8, \quad \sigma_Y = 1.2$
E) $\mu_Y = 5, \quad \sigma_Y = 1.2$

answer= E

Solution: We have

$$\mu_Y = E[Y] = -2E[X] + 30 = (-2)(12.5) + 30 = 5$$

and

$$\sigma_Y = \sqrt{V[Y]} = |-2|\sqrt{V[X]} = (2)\sqrt{0.36} = 1.2$$

11. Let X be a continuous random variable with the probability density function given by

$$f_X(x) = \begin{cases} kx^3, & 0 < x < 1; \\ 0, & \text{else where.} \end{cases}$$

What is the value of k?

A) 1 B) 3 C) 1/3 D) 4 E) 1/4

answer= D

Solution: Since $f_X(x)$ is a pdf, it must satisfy $\int_{-\infty}^{\infty} f(x)dx = 1$. Thus,

$$1 = \int_{-\infty}^{\infty} f(x)dx = \int_0^1 k\,x^3\,dx = \frac{k}{4} \quad \Rightarrow \quad k = 4$$

12. Suppose that a new car will require repair on the engine with probability 0.85, will require repair on the drive train with probability 0.37, and will require repair on both with probability 0.25. What is the probability that this car will require repair on neither engine nor drive train?

A) 0.121 B) 0.767 C) 0.786 D) 0.03 E) 0.05

answer = D

Solution: Consider the following events:
M: a new car will require repairs on the engine.
T: a new car will require repairs on the drive train.
We have that

$$P(M) = 0.85; \quad P(T) = 0.37; \quad P(M \cap T) = 0.25$$

We want to find

$$P(M' \cap T') = 1 - P(M \cup T) = 1 - [P(M) + P(T) - P(M \cap T)] = 1 - (0.85 + 0.37 - 0.25) = 0.03$$