École de gestion TELFER School of Management

LIÉE au LINKED with LEADERSHIP

ADM 2304
APPLIED STATISTICAL METHODS IN BUSINESS

9 February 2013, 9:00 – 11:00 AM

NAME (please print): ____________________________________

Student Number: __________________________ Section: _____________

Instructions

Length of Exam: 5 pages, plus 4 pages of Minitab output (please return).

Please show all your work and explain your answers briefly where required. All tests must include hypotheses, test statistics and rejection regions, decisions, and conclusions for full marks.

You are encouraged to use the Minitab output as much as possible.

You are permitted to have a non-programmable calculator and a double-sided sheet (8.5 x 11 inch) of notes.

Statistical tables (normal and t) are provided separately.

Marks:	________	+	________	+	_______	+	________	=	___________
	11		7		7		10		35

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ______________________________________

Question 1. [11 marks]

Having recognized that the high-tech sector has been a driver of economic growth, Industry Canada wants to make sure that technology companies receive sufficient bank loans. It collected information on bank loans provided to 10 technology companies and 8 retail companies. Industry Canada wants to know whether there are differences in the sizes of bank loans between the retail and the high-tech sectors.

Appendix A shows some analyses of the data. Note that the data have been sorted.

a) Explain the nature of the appropriate test. Should the test be based on independent or paired samples? Explain briefly.

[2]

b) Given your answer in (a), is the appropriate test a parametric or a non-parametric test? Please make reference to *specific* boxplots in your answer.

[2]

c) Notwithstanding your answer in (b), perform the appropriate parametric test to investigate whether the loan sizes are the same or different. Perform the test at the 5% level of significance. Use the critical value approach to define the rejection region.

[4]

d) Now perform an appropriate non-parametric test to investigate whether they are the same or different. Perform the test at the 5% level of significance.

[3]

Question 2. [7 marks]

An agroponics business delivers fresh vegetables to grocery stores. It offers a full refund on any shipment found to have excessive spoilage (at least 5% of the produce). Recently, spoilage levels have been high. In fact, 19% of shipments have been rejected by the stores and the refund claimed. The company examines its processes and considers that greater use of refrigerated trucks may reduce spoilage.

As a pilot project, refrigerated trucks are introduced on a selection of routes. A random selection of 140 deliveries are chosen and 16 deliveries are found to contain excessive spoilage.

(a) Because the trucks are expensive, the company will only introduce the trucks if spoilage is reduced by more than 4 percentage points to below 15%. Is there evidence that this is the case?

[4]

(b) Management decides to use the trucks and study the issue further. They would like to estimate the proportion of deliveries that will cost them a refund, to within a margin of error of ± 3 percentage points. Based on the first study (16 bad deliveries in 140), how many deliveries would they need to check to estimate the spoilage rate to obtain the given margin of error, with 95% confidence?

[2]

(c) For the above calculations, it is assumed that __________________________ has a normal distribution.

[1]

Question 3. [7 marks]

A major problem faced by a number of health authorities across Canada is that quite often the patient's stay in hospital is prolonged because the community services required in order for them to be cared for outside of the hospital are simply not available. These patients are called ALC ("alternative level of care") patients. A local health authority was interested in knowing whether there is a difference between the lengths of stay of ALC patients in the hospital depending on whether they were being transferred from the hospital to "Chronic Care" versus a "home for the aged". To that end, they collected two random samples of 60 patients from hospitals in the region. One sample consisted of the lengths of stay of patients who afterwards transferred to "Chronic Care" and the other sample consisted of the lengths of stay of patients who afterwards transferred to a "home for the aged".

Appendix B shows the distributions of the samples and their differences.

(a) Test whether there is a difference in length of stay depending on the location being transferred to at a significance level of 0.05. Assume that the population variances are equal.

[5]

(b) The test in (a) assumes that ______________________________ are normally distributed.

[1]

(c) An astute analyst noted that neither sample from part (a) contained lengths of stay longer than 40 days despite anecdotal evidence that many patients waited much longer. Further investigation determined that during the data collection, lengths of stay of longer than 40 days were discarded as outliers. Including these points led to two highly skewed data sets. If those lengths of stay were included, what test would be used to answer the question in (a) and why? Write down the hypotheses for the appropriate test.

[1]

Question 4. [10 marks]

The health authority produced a second set of samples but this time they wanted to take into account that another potentially major factor in the analysis could be the hospitals from which the patients are being transferred. They took two samples of size 45 (from the complete data set with all lengths of stay included) but this time they made sure that for each patient at hospital x who was transferred to "Chronic Care" there was also a patient from that same hospital in the other sample who was transferred to a "home for the aged". Appendix C shows the boxplots of the data and some partial analyses.

(a) Explain the nature of the appropriate test. Should the test be based on independent or paired samples? Explain briefly.

[2]

(b) Given your answer in (a), is the appropriate test a parametric or a non-parametric test? Please make reference to *specific* boxplots in your answer.

[2]

(c) Notwithstanding your answer in (b), perform an appropriate parametric test to determine whether there is sufficient evidence of a difference in the average lengths of stay at the 0.05 significance level. Use the p-value approach.

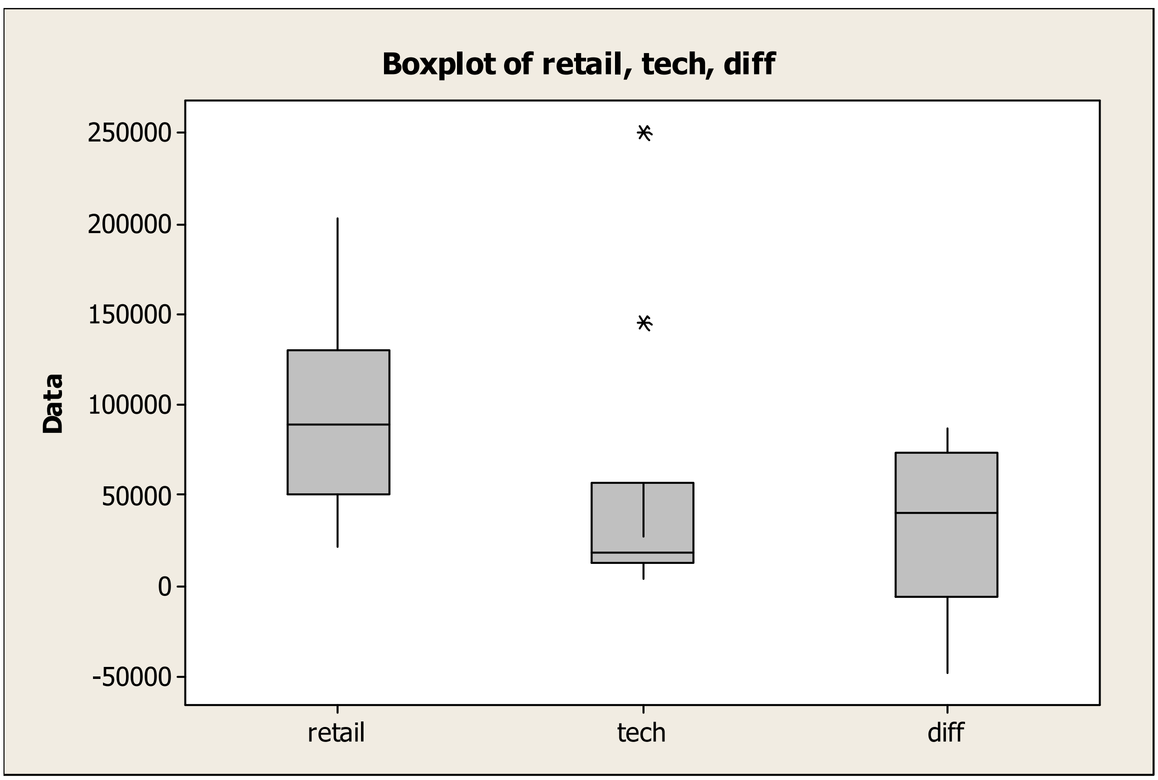
[4]

(d) Now calculate a 95% confidence interval for the average difference in hospital stays between transfers to “Chronic Care” versus a “home for the aged”.

[2]

Appendix A.

retail	tech	diff
*	4246	*
*	7344	*
22200	14780	7420
45000	15300	29700
68340	18000	50340
82620	18360	64260
96599	20400	76199
114000	27472	86528
135000	144840	-9840
202500	249997	-47497



Two-Sample T-Test and CI: loan, sector

```
Two-sample T for loan

sector   N   Mean  StDev  SE Mean
retail   8  95782  56289    19901
tech    10  52074  80772    25542


Difference = mu (retail) - mu (tech)
Estimate for difference:  43708.5
95% CI for difference:
T-Test of difference = 0 (vs not =): T-Value = _____  P-Value = _____  DF = 15
```

Paired T-Test and CI: retail, tech

```
Paired T for retail - tech

            N     Mean    StDev  SE Mean
retail      8  95782.4  56288.5  19901.0
tech        8  63643.6  87307.0  30867.7
Difference  8  32138.8  46265.7  16357.4


95% CI for mean difference:
T-Test of mean difference = 0 (vs not = 0): T-Value = _____  P-Value = _____
```

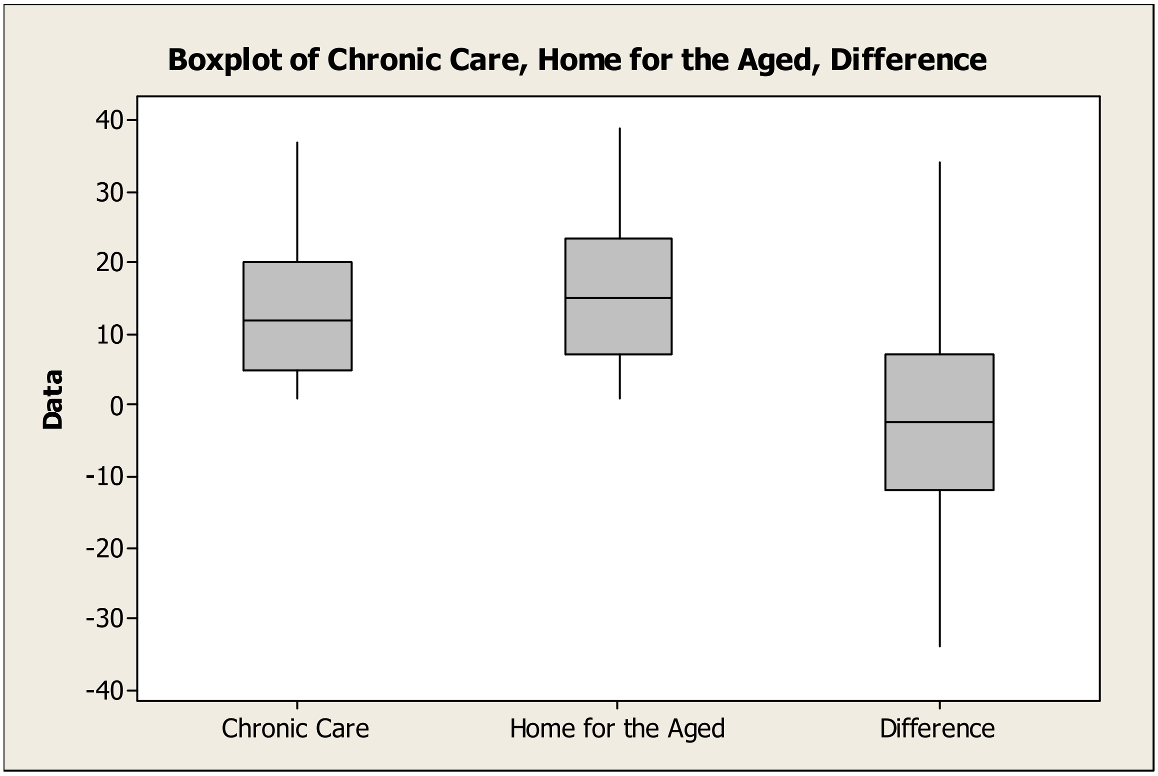
Mann-Whitney Test and CI: retail, tech

```
         N  Median
retail   8   89610
tech    10   18180


Point estimate for ETA1-ETA2 is 61608
95.4 Percent CI for ETA1-ETA2 is (3840,106656)
W = 100.0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.0368
```

Wilcoxon Signed Rank Test: diff

```
Test of median = 0.000000 versus median not = 0.000000

                N
              for   Wilcoxon         Estimated
      N  N*  Test  Statistic      P     Median
diff  8   2     8       30.0  0.107      34510
```

Appendix B.



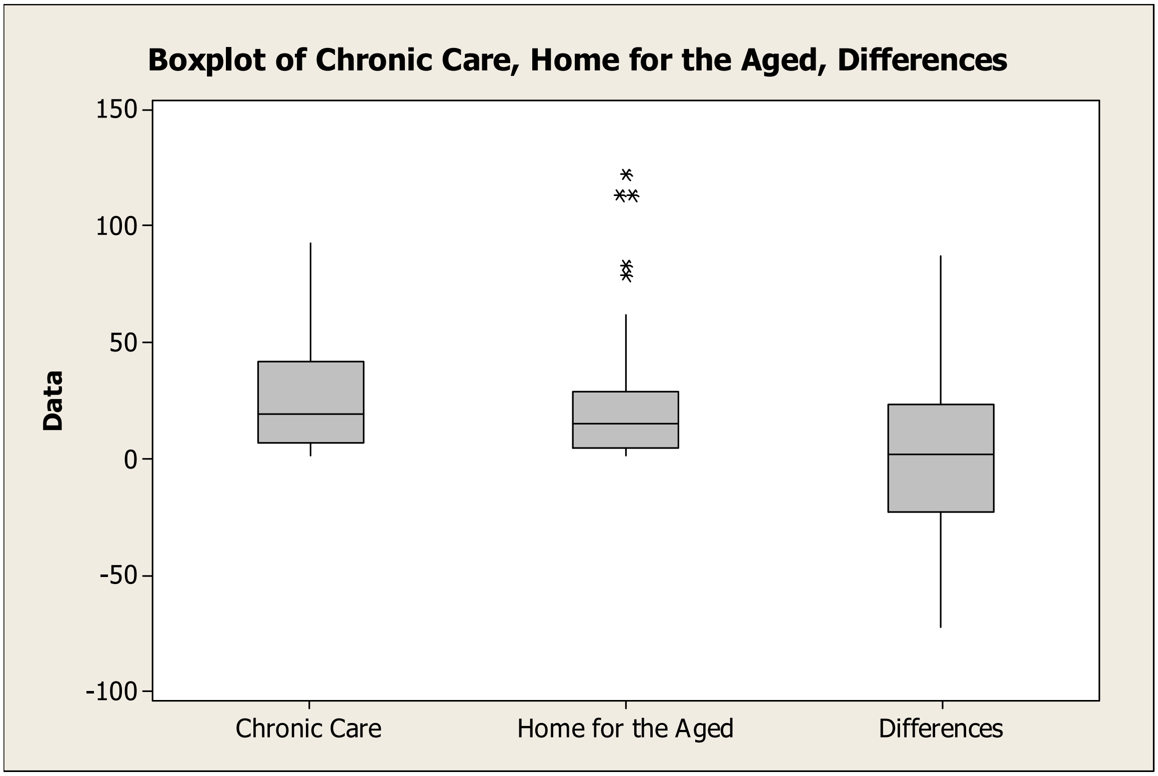
Two-Sample T-Test and CI: Chronic Care, Home for the Aged

```
Two-sample T for Chronic Care vs Home for the Aged

                    N  Mean  StDev  SE Mean
Chronic Care       60  13.9   10.4      1.3
Home for the Aged  60  15.9   10.8      1.4


Difference = mu (Chronic Care) - mu (Home for the Aged)
Estimate for difference:  -2.05
95% CI for difference:
T-Test of difference = 0 (vs not =): T-Value = ____  P-Value = _____ DF = ___
```

Paired T-Test and CI: Chronic Care, Home for the Aged

```
Paired T for Chronic Care - Home for the Aged

                    N   Mean  StDev  SE Mean
Chronic Care       60  13.88  10.41     1.34
Home for the Aged  60  15.93  10.81     1.40
Difference         60  -2.05  15.16     1.96

95% CI for mean difference:
T-Test of mean difference = 0 (vs not = 0): T-Value = ____  P-Value = ____
```

Appendix C.



Descriptive Statistics:

	N	Mean	StDev	SE Mean
Chronic Care	45	27.44	24.81	3.70
Home for the Aged	45	41.44	31.57	4.71
Difference	45	-14.00	34.39	5.13

Wilcoxon Signed Rank Test: Differences

Test of median = 0.000000 versus median not = 0.000000

	N	N for Test	Wilcoxon Statistic	P	Estimated Median
Differences	45	45	277.0	0.007	-14.00

Mann-Whitney Test and CI: Chronic Care, Home for the Aged

	N	Median
Chronic Care	45	19.00
Home for the Aged	45	30.00

Point estimate for ETA1-ETA2 is -12.00
95.0 Percent CI for ETA1-ETA2 is (-19.00,-4.00)
W = 1707.5
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.0062
The test is significant at 0.0061 (adjusted for ties)

Standard Normal Distribution

P(Z < z)   (z negative)

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	**z**
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) **(z positive)**

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

t_α

$\alpha = P(t > t_\alpha)$ = one-tail probability

df	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73