MATH2004A- Test 1: 16:35–17:25, Oct 3

Surname ____________ **First Name** ____________**Student #** ________

Total: 15 points. No partial marks for Questions 1-4.
Closed book! Non-programmer calculators are allowed!

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1. (1 point) Find the area of the triangle with vertices $P(-3,2,3)$, $Q(-2,2,1)$, and $R(-3,1,1)$.

   (a) 1 (b) 3/2 (c) 7/2 (d) 5/2 (e) 1/2

   Solution: (b). $\vec{PQ} = Q - P = (1,0,-2)$, $\vec{PR} = R - P = (0,-1,-2)$. $A = \frac{1}{2}||\vec{PQ} \times \vec{PR}|| = \frac{1}{2}||(-2,2,-1)|| = 3/2$.

2. (1 point) Which of the following is the intersection of the plane $x + y + z = 2018$ and the line $(x,y,z) = (1,-1,1) + t(1,1,-2),\ t \in \mathbb{R}$?

   (a) $(-6,-4,4)$ (b) $(-4,-3,4)$ (c) $(-1,2,4)$ (d) $(3,1,4)$ (e) no intersection

   Solution: (e). Note that $(1,1,1)\cdot(1,1,-2) = 0$, and the point $(1,-1,1)$ is not on the plane, the plane and the line are parallel, and no intersection.

3. (1 point) Find the cosine of the angle between two planes $2x - y - 2z = 3$ and $3x - 4y = 1$.

   (a) $\frac{2}{15}$ (b) $-\frac{2}{15}$ (c) $\frac{2}{3}$ (d) 1.2 (e) 0.7

   Solution: (c).
   $\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|} = \frac{(2,-1,-2)\cdot(3,-4,0)}{|(2,-1,-2)|\,|(3,-4,0)|} = \frac{10}{3(5)}$.

4. (1 point) Convert the point $(8, \frac{\pi}{6})$ from polar coordinates to Cartesian coordinates (x,y), then $x =$

   (a) 0 (b) 1 (c) 2 (d) 4 (e) $4\sqrt{3}$

   Solution: (e). $x = r\cos\theta = 8\cos\frac{\pi}{6} = 4\sqrt{3}$.

5. (3 points) At $t = 2$, find the tangent line to the curve $x = t^3 + t^2, y = t^4 + 16t$.

   Solution: At first we need the slope of the tangent line.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3 + 16}{3t^2 + 2t}.$$

1 point

When $t = 2$, the slope of the tangent line is 3. The point is $(12, 48)$.

1 point

The tangent line is: $y = 3x + b$. Sub $(12, 48)$, $b = 12$. Thus $y = 3x + 12$.

1 point

6. (4 points) Find the area of the closed region enclosed by $x = t^3 - t$, $y = t^3 + t^2 - 2t$, $0 \le t \le 1$.

Solution:

$$A = |\int_{\alpha}^{\beta} y(t)x'(t)dt|$$

(1 point)

$$= \left| \int_0^1 (t^3 + t^2 - 2t)(t^3 - t)'dt \right| = |\int_0^1 (3t^5 + 3t^4 - 7t^3 - t^2 + 2t)dt|$$

$$= |\left[\frac{1}{2}t^6 + \frac{3}{5}t^5 - \frac{7}{4}t^4 - \frac{1}{3}t^3 + t^2\right]_0^1 | = \frac{1}{60}.$$

(3 points)

7. (4 points) Find the length of the polar curve: $r = \theta^2$, $0 \le \theta \le \sqrt{5}$.

Solution:

$$L = \int_{\alpha}^{\beta} \sqrt{(r')^2 + r^2} d\theta$$

1 point

$$= \int_0^{\sqrt{5}} \sqrt{(2\theta)^2 + (\theta^2)^2} d\theta = \int_0^{\sqrt{5}} \theta\sqrt{4 + \theta^2} d\theta$$

1 point

$$\int_4^9 \frac{1}{2}\sqrt{x}dx, \quad x = 4 + \theta^2$$

$$= \frac{1}{3}x^{3/2}|_4^9 = \frac{27 - 8}{3} = \frac{19}{3}.$$

2 points