Practice Midterm 2 solutions

1.

(a) $\det(A) = 48 \neq 0$, so matrix A is invertible.

(b)

$$A^{-1} = \begin{bmatrix} \frac{-1}{4} & 0 & 0 \\ 0 & \frac{-1}{6} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{-1}{6} \end{bmatrix}.$$

(c) x=$\begin{bmatrix} \frac{-3}{2} \\ 1 \\ 1 \end{bmatrix}$.

(d,e) Eigenvalues $\lambda = -4, 6, -2$. Eigenvectors corresponding to $\lambda = -4$ are

$$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r, r \in \mathbb{R}, r \neq 0$$

2.

(a) Unique solution: $ab \neq 5$

(b) Infinitely many solutions: $b = 2$ and $a = \frac{5}{2}$.

(c) No solutions: $ab = 5$ and $b \neq 2$.

3. $s \begin{bmatrix} -17 \\ 10 \\ 1 \end{bmatrix} + \begin{bmatrix} 12 \\ -7 \\ 0 \end{bmatrix}$, $s \in \mathbb{R}$. (Note: s can be zero since this isn't an eigenvector.)

4. $2e^{2t} \begin{bmatrix} 5 \\ 1 \end{bmatrix} - e^{-4t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

5.(b) $\begin{bmatrix} 1+i \\ 1 \end{bmatrix}$

6.

(a) $\{(x, y) \in \mathbb{R}^2 | y < x - 4\}$

(b) $\{f \in \mathbb{R} | f > 0\}$

(d) For $k = \frac{1}{2}, 1, 2, 3$