

MAT1322  
Some Practice For Test 3  
(a midterm from 2018)

Note: there are only 3 questions in this midterm that are on topics that we had not yet covered before Test 2, so this midterm from 2018 is not representative of what topics your Test 3 will cover...

SOLUTIONS  
TO SELECT  
EXERCISES

# Part I : Multiple-choice Questions:

[4 × 2 = 8 points]

Circle the correct answer. No justification is required.

## Question 1.

[2 points]

A potato with a temperature of 20° is put in a 200° oven and heats up according to the differential equation

$$\frac{dH}{dt} = k(H - 200)$$

where  $H(t)$  represents the potato's temperature  $t$  minutes after being placed in the oven. After 30 minutes the temperature of the potato is 120°C. What is the value of  $k$  to 3 decimal places?

- A.  $k = 1.120$       B.  $k = -0.027$       C.  $k = 0.027$       D.  $k = -1.120$       E.  $k = -0.015$

$$\int \frac{1}{H-200} dH = \int k dt$$

$$\Rightarrow \ln|H-200| = kt + c$$

$$\Rightarrow |H-200| = e^{kt+c}$$

$$\Rightarrow H-200 = Ae^{kt}$$

where  $A = \pm e^c$

We know  $H(0) = 20$

$$\Rightarrow 20 - 200 = Ae^0$$

$$\Rightarrow -180 = A$$

and  $H(30) = 120$

$$\Rightarrow 120 - 200 = -180e^{k(30)}$$

$$\Rightarrow -80 = -180e^{30k}$$

$$\Rightarrow \frac{80}{180} = e^{30k}$$

$$\Rightarrow k = \frac{\ln(80/180)}{30} \approx -0.027$$

## Question 2.

[2 points]

For all  $n \geq 1$ , let  $a_n = \frac{2n+1}{4n+2}$ . Which of the following statements is true?

A. The sequence  $\{a_n\}$  converges and the series  $\sum_{n=1}^{\infty} a_n$  also converges.

B. The sequence  $\{a_n\}$  converges but the series  $\sum_{n=1}^{\infty} a_n$  diverges.

C. The sequence  $\{a_n\}$  diverges but the series  $\sum_{n=1}^{\infty} a_n$  converges.

D. The sequence  $\{a_n\}$  diverges and the series  $\sum_{n=1}^{\infty} a_n$  also diverges.

**Question 3.**

[2 points]

Determine the sum of the geometric series  $\sum_{n=1}^{\infty} 9 \frac{5^{n-1}}{2^{2n-2}}$  if it exists.

- A. 15      B.  $\frac{3}{5}$       C. 1      D.  $\frac{12}{5}$       E. 3      F. The series is divergent.

**Question 4.**

[2 points]

What is the radius of convergence for  $\sum_{n=0}^{\infty} \frac{5^n (x-3)^n}{\sqrt{n+5}}$  ?

- A.  $R = \frac{1}{5}$       C.  $R = \frac{1}{8}$       E.  $R = \frac{3}{8}$       G.  $R = \frac{3}{5}$   
 B.  $R = \frac{1}{6}$       D.  $R = \frac{8}{5}$       F.  $R = \frac{2}{5}$       H. None of these.

Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{5^{n+1} (x-3)^{n+1}}{\sqrt{n+1+5}} \bigg/ \frac{5^n (x-3)^n}{\sqrt{n+5}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{5(x-3)\sqrt{n+5}}{\sqrt{n+6}} \right|$$

$$= 5|x-3| \qquad 5|x-3| < 1 \Rightarrow |x-3| < \frac{1}{5} \therefore R = \frac{1}{5}$$

## Part II : Long-Answer Questions:

[15 points]

Give detailed solutions, clearly showing each of your steps.

### Question 5.

[6 points]

Use an appropriate test method to determine whether each of the following series is convergent or divergent. State the conditions needed in order for the test to apply.

(a) 
$$\sum_{n=1}^{\infty} \frac{n^3 + 2}{\sqrt{n^7 + 8}}$$

(b) 
$$\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^2}$$

(c) 
$$\sum_{n=1}^{\infty} \left( \frac{2n + 1}{5n} \right)^n$$

**Question 6.**

[4 points]

(a) (2 pts) Does the following series converge:  $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 4}$  ? Justify your answer.

(b) (1 pt) Does the series above converge absolutely? Please justify your answer.

(c) (1 pt) Give an upper bound on the absolute value of the error if we use the first 1000 terms of the series to approximate its sum.

You do NOT need to compute  $S_{1000}$ . Just give the maximum possible error.

### Question 7.

[5 points]

(a) (4 pts) Find the power series representation for  $\ln(3-x)$  and determine interval of convergence.

hint:  $\ln(3-x) = -\int \frac{1}{3-x} dx$ . Do NOT forget to find the constant of integration.

Using  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$  we have

$$\frac{1}{3-x} = \frac{1}{3(1-\frac{x}{3})} = \frac{1}{3} \left( \frac{1}{1-\frac{x}{3}} \right) = \frac{1}{3} \sum_{n=0}^{\infty} \left( \frac{x}{3} \right)^n = \frac{1}{3} \sum_{n=0}^{\infty} \frac{x^n}{3^n} = \sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$$

which is good for  $|\frac{x}{3}| < 1$  or  $|x| < 3$  ( $R=3$ )

Using  $\ln(3-x) = -\int \frac{1}{3-x} dx$  we get

$$\ln(3-x) = -\int \sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}} dx = C - \sum_{n=0}^{\infty} \frac{x^{n+1}}{3^{n+1}(n+1)}$$
 which is still good for  $|x| < 3$  or  $-3 < x < 3$

Use  $x=0$  to solve for  $C$ :  $\ln(3-0) = C - \sum_{n=0}^{\infty} \frac{0^{n+1}}{3^{n+1}(n+1)} \therefore C = \ln(3)$

length of interval is  $3 - (-3) = 6$   
 $\Rightarrow R = \frac{6}{2} = 3$

$\therefore$  the power series representation for  $\ln(3-x)$  is

$$\ln(3-x) = \ln 3 - \sum_{n=0}^{\infty} \frac{x^{n+1}}{3^{n+1}(n+1)}$$

Endpoints?

when  $x=-3$ , we get  $\ln 3 - \sum_{n=0}^{\infty} \frac{(-3)^{n+1}}{3^{n+1}(n+1)} = \ln 3 - \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$  which converges by AST.

$x=3$  is not in the domain of  $\ln(3-x)$

$\therefore$  the interval of convergence is  $-3 \leq x < 3$  and radius is  $R=3$

(b) (1 pt) Use part (a) to express  $\ln 2$  as the sum of an infinite series.

$\ln(2) = \ln(3-1)$  so we can plug in  $x=1$ :

$$\ln(2) = \ln 3 - \sum_{n=0}^{\infty} \frac{1}{3^{n+1}(n+1)}$$

(additional page for scrap work)

(end of Test 2 – happy  $\pi$  day!)