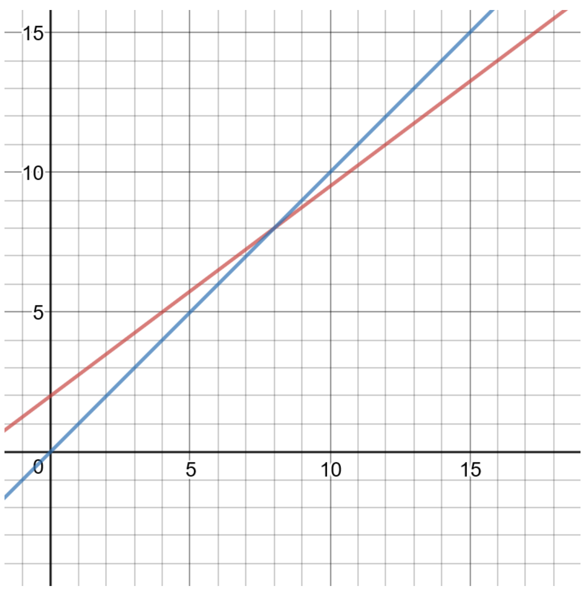
# MAT1330 - DGD3

**3.2 like #6.** Given the DTDS governing the daily dose of a drug, $M_{t+1} = 0.75M_t + 2$, do three iterations of a cobweb starting at $M_0 = 16$ mg/L. Then plot the solution you found this way on a graph of $t$ vs $M_t$.

**Tips:** *Draw the cobweb next to the graph on which you'll transcribe the solution, as on page 131 and 132 of the textbook. Label your points to help students see the connections.*

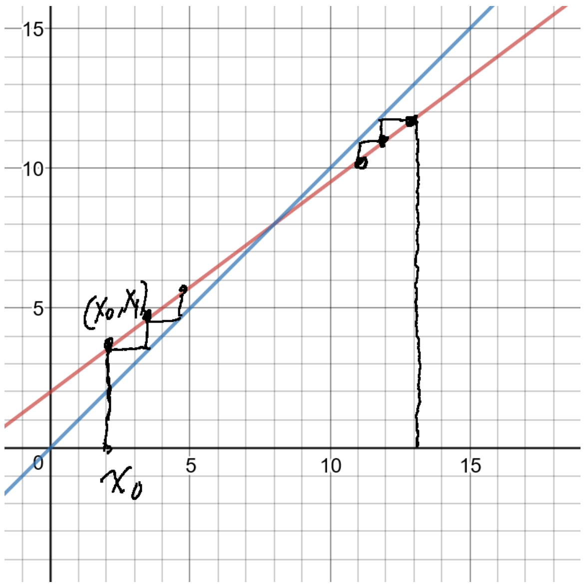
*Solution:* (cobweb)




The general solution formula for a linear DTDS with updating function $f(M) = 0.75M + 2$ is $M_t = (0.75)^t(M_0 - M^*) + M^*$ where $M^* = \frac{2}{1-0.75} = 8$. Here, $M_0 = 16$, so the formula is

$$M_t = (0.75^t)(16 - 8) + 8 = 8(0.75^t) + 8.$$

*Date*: January 26, 2019.




**2.** Find the updating function underlying the DTDS $x_{t+1} = \frac{x_t}{x_t - 1}$. Solve for all fixed points.

*Solution:*

The updating function is

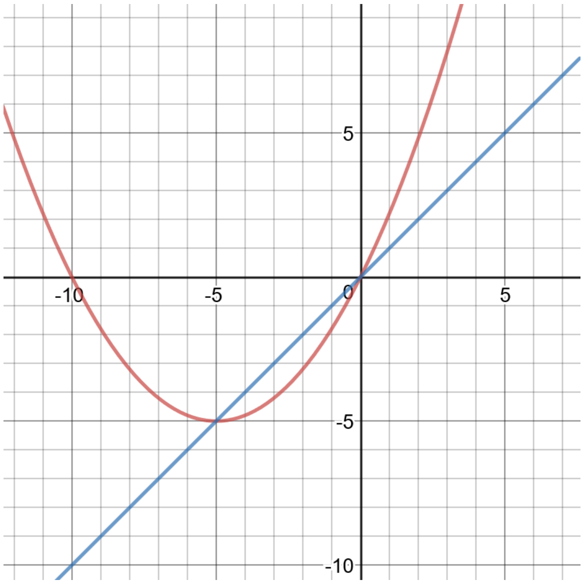
$$f(x) = \frac{x}{x-1}.$$

To find fixed pints, we let

$$f(x) = x, \quad \frac{x}{x-1} = x, \quad x = 0, 2.$$

**3.** Suppose $x_{t+1} = -\frac{1}{5}x_t^2 + 2x_t$. Cobweb this DTDS starting at $x_0 = 2$. Find all fixed points and classify them according to their stability.

*Solution:* The fixed points are $x^* = 0$ and $x^* = 5$. Cobwebbing shows 0 is unstable but 5 is stable.




**3.3 like #6.** *Solution:* $p_0 = 100$, $p_4 = 400$. Note that

$$p_4 = p_0 r^4, \Rightarrow 400 = 100r^4, \Rightarrow r^4 = 4, \Rightarrow r = \sqrt{2}.$$

**3.3 like #7.** *Solution:* $p_0 = 1200$, $p_{25} = 400000$. Note that

$$p_{25} = p_0 r^{25}, \Rightarrow 400000 = 1200r^{25}, \Rightarrow r^{25} = 1000/3, \Rightarrow r = 1.26158.$$

**3.3 like #20.** *Solution:* $b_0 = 10^6$, $b_t = 10^7$, $r = 1.5$. Note that

$$b_t = b_0 r^t, \Rightarrow 10^7 = 10^6(1.5)^t, \Rightarrow 1.5^t = 10, \Rightarrow t = 1/log1.5 = 5.67887.$$

**3.4 like #12.** *Solution:* Let

$$x = \frac{4x}{4x + 0.5(1-x)}, \Rightarrow x = 0, 1.$$

**limits 1.** Let

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x > 2 \\ 3k + 6 & \text{if } x = 2 \end{cases}$$

For what values of the constant $c$ does the limit $\lim_{x\to 2} f(x)$ exist?

*Solution:* Since

$$\lim_{x\to 2^-} f(x) = 4c + 4, \quad \lim_{x\to 2^+} f(x) = 8 - 2c.$$

When the limit $\lim_{x\to 2} f(x)$ exists, $4c + 4 = 8 - 2c, \Rightarrow c = \frac{2}{3}$. $\lim_{x\to 2} f(x) = \frac{20}{3}$.

**limits 2.** Calculate the limit: $\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x^2-1}$.

*Solution:*

$$\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x^2-1} = \lim_{x\to 1} \frac{(\sqrt{x+8}-3)(\sqrt{x+8}+3)}{(x-1)(x+1)(\sqrt{x+8}+3)} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)(\sqrt{x+8}+3)}$$
$$= \lim_{x\to 1} \frac{1}{(x+1)(\sqrt{x+8}+3)} = \frac{1}{12}.$$

**limits 3.** Calculate the following limits. If a limit does not exist, determine if it is $+\infty$, $-\infty$, or neither.

a) $\lim_{x\to -\infty} \frac{2x^2-x+3}{x+4}$;

b) $\lim_{x\to\infty} [\ln(x+2) - \ln(2x)]$;

c) $\lim_{x\to\infty} \arctan(x^2 - x^4)$;

*Solution:* a) Let $t = x+4$.

$$\lim_{x\to -\infty} \frac{2x^2-x+3}{x+4} = \lim_{t\to -\infty} \frac{2(t-4)^2-(t-4)+3}{t} = \lim_{t\to -\infty} \frac{2(t^2-8t+16)-t+7}{t}$$
$$= \lim_{t\to -\infty} \frac{2t^2-17t+39}{t} = \lim_{t\to -\infty} (2t-17+\frac{39}{t}) = -\infty.$$

b) $\lim_{x\to\infty} [\ln(x+2) - \ln(2x)] = \lim_{x\to\infty} [\ln(x+2) - \ln x - \ln 2]$

$= \lim_{x\to\infty} [\ln \frac{x+2}{x} - \ln 2] = \ln 1 - \ln 2 = -\ln 2$;

c)

$$\lim_{x\to\infty} \arctan(x^2 - x^4) = \arctan \lim_{x\to\infty} \left[x^4(\frac{1}{x^2} - 1)\right] = \arctan(-\infty) = -\frac{\pi}{2}.$$