MAT1330 DGD2

2.3 # 68. Graph the function. Give the average, max, min, amplitude, period and phase of each and mark them on the graph:

$$f(x) = 3 + 4\cos(2\pi\left(\frac{x-1}{5}\right)).$$

Solution: For

$$M + A\cos(\frac{2\pi}{T}(x-\varphi))$$

where M is the mean=average, A is the amplitude, T is the period, and φ is the phase. So rewriting our function slightly we get

$$3 + 4\cos(\frac{2\pi}{5}(x-1))$$

which is in the correct form to read off the values. For the graph: the mean is the horizontal center line; the amplitude is how far above and below the center line the graph will oscillate; the period is the distance between adjacent peaks; and the phase is how far right the first peak is shifted away from 0. Check your answer with a graphing app.

Tips: *Although the inverse sine and cosine functions are in this section, we are deferring their discussion until much later in the course. Students may never have seen the above description of oscillating functions before, but it is incredibly useful in applications to the physical and life sciences, so very much worth their understanding.*

Simplify. $\cos(\arcsin(\frac{a}{b}))$.

Solution: $\cos(\arcsin(\frac{a}{b})) = \sqrt{1-\sin^2(\arcsin(\frac{a}{b}))} = \sqrt{1-\frac{a^2}{b^2}}$.

3.1 #2. Write the updating function associated with each DTDS, and evaluate it at the given arguments. Which are linear?

$$m_{t+1} = \frac{m_t^2}{m_t+2}; \quad \text{Evaluate } f \text{ at } m_t = 0, m_t = 8, m_t = 20.$$

Solution: Let's say m_t represents mass of a growing fungus in kg and t is days. The updating function is the function f such that $m_{t+1} = f(m_t)$. So here,

$$f(x) = \frac{x^2}{x+2}$$

is the updating function. It is not linear. We have

$f(0) = 0$ Meaning: if we start with 0 fungus, we have 0 fungus after a day.

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$$f(8) = \frac{8^2}{8+2} = 6.4 \qquad \text{Meaning: if we start with 8kg fungus, we have only 6.4kg after a day.}$$

and similarly $f(20) = 400/22 \simeq 18.2$.

3.1 #12. Find the backward DTDS associated to each of the following DTDS. Use it to find the value at the previous time:

$$M_{t+1} = 0.75M_t + 2, \quad \text{Find } M_0 \text{ if } M_1 = 16.$$

Solution: The forward DTDS is given. The backward DTDS means solving for M_t in terms of M_{t+1}:

$$0.75M_t = M_{t+1} - 2 \Leftrightarrow M_t = \frac{4}{3}(M_{t+1} - 2).$$

(Here, we used that $0.75 = \frac{3}{4}$, so dividing by 0.75 is the same as multiplying by $\frac{4}{3}$.)

The inverse updating function is thus $f(x) = \frac{4}{3}(x - 2)$. So if $M_1 = 16$, then

$$M_0 = f(M_1) = \frac{4}{3}(16 - 2) = \frac{4}{3}(14) = \frac{56}{3} \simeq 18.67.$$

We check by using the original system: if you plug in $\frac{56}{3}$ for M_0, you get $M_1 = \frac{3}{4}(\frac{56}{3}) + 2 = 14 + 2 = 16$. (Or use the rounded value, and get close).

Tips: *On MapleTA, they will often be asked to give the exact answer, as a fraction, so use the opportunity to have them practice with fractions. Be patient and kind in your explanations!*

3.1 #18 and 22. Graph some values of the DTDS, starting with the given initial condition. Write down a formula for the general solution and sketch its graph. Sketch also the graph of the updating function. $\ell_{t+1} = \ell_t - 1.7$, with initial value $\ell_0 = 13.1$cm.

Tips: *For this question, try to convince students of the general solution ℓ_t by doing the iterations algebraically, as below, for as many steps as it takes for them to see the pattern. You can then compare with the general solution of a linear DTDS, as presented in class.*

Solution: We compute:

$$\ell_0 = 13.1, \ell_1 = 11.4, \ell_2 = 9.7, \ell_3 = 8.0, \ell_4 = 6.3, \ell_5 = 4.6.$$

Plot these on a graph of t versus ℓ.

To find the general solution, we iterate:

$$\ell_1 = \ell_0 - 1.7, \ell_2 = \ell_1 - 1.7 = (\ell_0 - 1.7) - 1.7 = \ell_0 - 2(1.7).$$

Repeating this reveal that the general solution is in fact just

$$\ell_t = \ell_0 - 1.7t.$$

We plug in $\ell_0 = 13.1$, which gives the solution with our initial condition: $\ell_t = 13.1 - 1.7t$. We plot the graph of this function, which is a straight line with slope -1.7 passing through each of the points graphed previously.

The updating function was $f(x) = x - 1.7$. This is also a straight line, but with positive slope; this is certainly not the same function as the general solution!

3.1 #19 and 23. Same, but with $n_{t+1} = 0.5n_t$ and $n_0 = 1200$.

Tips: *This is similar to a homework problem. Again, try to help student identify the pattern rather than immediately using the formula from class.*

Solution: We compute

$$n_0 = 1200, n_1 = 600, n_2 = 300, n_3 = 150, n_4 = 75, n_5 = 37.5.$$

Plot these on a graph of t versus n.

To find the general solution, we iterate:

$$n_1 = 0.5n_0; n_2 = 0.5n_1 = 0.5(0.5n_0) = (0.5)^2 n_0; n_3 = 0.5n_2 = 0.5(0.5^2 n_0) = 0.5^3 n_0$$

and so on, which gives the general solution

$$n_t = (0.5)^t n_0.$$

With initial condition $n_0 = 1200$, we get the solution $n_t = 1200(0.5)^t$. Graph this; it is a decreasing exponential function going through our points.

The updating function is $f(x) = 0.5x$, which is a straight line. Again, the updating function is certainly not the solution.

3.1 #20 and 24. Same, but with $M_{t+1} = 0.75M_t + 2$, and $M_0 = 16$.

Tips: *This is the most general kind we consider in this chapter.*

Solution: We compute

$$M_0 = 16, M_1 = 14, M_2 = \frac{25}{2}, M_3 = 11.375, M_4 = 10.53125, M_5 = 9.8984375.$$

Plot these on a graph of t versus M.

To find the general solution, we iterate:

$$\begin{aligned} M_1 &= 0.75M_0 + 2 \\ M_2 &= 0.75M_1 + 2 = 0.75(0.75M_0 + 2) + 2 = 0.75^2 M_0 + 0.75 * 2 + 2 \\ M_3 &= 0.75M_2 + 2 = 0.75^3 M_0 + 0.75^2 * 2 + 0.75 * 2 + 2 \end{aligned}$$

which, by using the geometric series, gives

$$M_t = 0.75^t M_0 + 2\left(\frac{1 - 0.75^t}{1 - 0.75}\right) = 0.75^t M_0 + 8(1 - 0.75^t) = 0.75^t(M_0 - 8) + 8.$$

We compare with the general formula for the solution of the linear DTDS with updating function $f(x) = ax + b$, which is to set $M^* = \frac{b}{1-a}$ and then $M_t = a^t(x_0 - x^*) + x^*$. Here, $f(x) = 0.75x + 2$ so $a = 0.74$, $b = 2$ and $M^* = \frac{2}{1-0.75} = 8$, giving the answer above.

Again, the graph of the solution (with $M_0 = 16$) passes through our points and is an exponential; but the graph of the updating function is linear.

Other questions. If you have time, or if students are asking for an application, then you could work through textbook Example 3.1.4 + Example 3.1.9.