MATH1107E Test #2

Tuesday, March 19, 2019 1:42 PM

Carleton University
School of Mathematics and Statistics
Instructor: Ahmed Almaskut
/38 Marks

1. (10 marks) Solve the following system of linear equations using the inverse method:

$$x - 2y - 3z = -1$$
$$x - y - 2z = 1$$
$$-x + 3y + 5z = 2$$

in matrix form:

$$A = \begin{bmatrix} 1 & -2 & -3 \\ 1 & -1 & -2 \\ -1 & 3 & 5 \end{bmatrix}$$

$$M = \left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 1 & 0 & 0 \\ 1 & -1 & -2 & 0 & 1 & 0 \\ -1 & 3 & 5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 - R_1 \\ R_3 = R_3 + R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 - R_2 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 7 & -3 & 3 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 + 3R_3 \\ R_2 = R_2 - R_3 \\ R_3 = R_3 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 0 & 2 & -1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 + 2R_2 \\ R_2 = R_2 \\ R_3 = R_3 \end{array}$$

$Ax = b$, $x = A^{-1}b$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

The solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

The solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

2. (5 marks) Determine the value(s) of h such that the matrix is the augmented matrix for a consistent linear system.

   i. (3 mark)

$$\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & h \\ 0 & 0 & 7-2h \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 - 2R_1 \end{array}$$

for the matrix to be consistent $h \neq \frac{7}{2}, h \in \mathbb{R}$

   ii. (2 marks)

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \sim \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & h+15 & 3 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 - 5R_1 \end{array}$$

$h + 15 = 3 \Rightarrow h = -12$

for this matrix to be consistent $h \neq -12$.

3. (12 marks) Find the inverse of the following matrix, if it exists. If you determine that an inverse does not exist, explain why.

   i. (8 marks)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 4R_1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 + R_3 \\ R_3 = R_3 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 0 & -1 & -6 & 1 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 + 2R_2 \\ R_2 = R_2 \\ R_3 = R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_3 \\ R_3 = R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = -R_3 \end{array} \quad \text{So} \quad A^{-1} = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$

Check: $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   ii. (4 marks)

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$$A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ -1 & 5 & 6 & 0 & 1 & 0 \\ 5 & -4 & 5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & 6 & 10 & -5 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 + R_1 \\ R_3 = R_3 - 5R_1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & 0 & 0 & -7 & -2 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 - 2R_2 \end{array}$$

The inverse of A does NOT exist because through row reduction you get a row of zero which means matrix A cannot be reduced the identity matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

4. (11 marks) Find the rank for the given coefficient matrix for a system of linear equations.

i. (7 marks)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 + 2R_1 \\ R_3 = R_3 - R_1 \end{array} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -3 \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 - R_2 \end{array} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = -\frac{1}{3} R_3 \end{array}$$

rank of $A = 3$

ii. (4 marks)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & -1 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & -1 & -4 \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 - 3R_1 \\ R_3 = R_3 + R_1 \end{array} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_1 = R_1 \\ R_2 = R_2 \\ R_3 = R_3 + R_2 \end{array}$$

$\therefore$ The rank of A is 2.