PART A (35 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark 1. Let $\mathbf{u} = (5, -2, 4)$ and $\mathbf{v} = (1, 3, -2)$. Find $\mathbf{u} - 2\mathbf{v}$.

A: $(6, 1, 2)$	B: $(3, 8, -8)$	C: $(7, -8, 7)$	D: $(3, -8, 8)$	E: $(4, -5, 6)$

1 mark 2. Let $\mathbf{u} = (1, 2, -2)$, $\mathbf{v} = (3, 0, 1)$ and $\mathbf{w} = (2, 2, -1)$. Find the volume of the parallelepiped with edges (i.e. determined by) $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

A: 0	B: -4	C: 4	D: 6	E: -6

1 mark 3. Which one of the following vectors is perpendicular to the line shown here?

$$(x, y, z) = (1 - t)(1, 2, -3) + t(3, -7, 4)$$

A: $(3, -7, 4)$	B: $(1, 2, -3)$	C: $(4, -5, 1)$	D: $(4, 0, -3)$	E: $(-7, 0, 2)$

1 mark 4. Find the line through the point $(-1, 5)$ and parallel to the line shown here.

$$(x, y) = (2, -1) + t(-1, 3)$$

A: $3x + y = 2$	B: $x - 3y = 2$	C: $x - 5y = 0$	D: $5x + y = 0$	E: $2x - y = -7$

1 mark 5. If (a, b, c) is the point of intersection of the lines $\mathbf{x}(t) = (2, 5, -1) + t(-1, 2, 3)$ and $\mathbf{x}(s) = (3, 2, -6) + s(2, -3, -4)$, find c.

A: 3	B: 5	C: -1	D: -10	E: 0

1 mark 6. Let B be the row-reduced echelon form of the matrix A shown below. Find the first row of B.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

A: $[\,1 \;\; 1 \;\; 2 \;\; 1\,]$	B: $[\,1 \;\; 0 \;\; 0 \;\; 0\,]$	C: $[\,1 \;\; 1 \;\; 0 \;\; 1\,]$
D: $[\,1 \;\; 0 \;\; -1 \;\; 2\,]$	E: $[\,1 \;\; 0 \;\; -1 \;\; 0\,]$	

1 mark 7. Find the value(s) of k for which the following system of linear equations has infinitely many solutions.

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 3x_1 - x_2 + x_3 &= 0 \\ x_1 - 3x_2 + kx_3 &= -2 \end{aligned}$$

A: $k = 0$ only	B: any value except $k = 3$	C: $k = -3$ only
D: $k = 3$ only	E: no value of k	

1 mark 8. Find the solution to the system of linear equations with the augmented matrix shown here.

$$\left[\begin{array}{ccccc|c} 2 & 4 & 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 0 & 9 & 12 \end{array}\right]$$

A: $(2,4,3,1,2)$	B: $(8-2s-4t, s, 4-3t, 3-2t, t)$	C: $(6-s+2t, 4-3s, 3-2t, t)$
D: $(8-2t, t, 4, 3t, 1)$	E: no solution	

1 mark 9. If F is a 2×4 matrix, G is a 3×3 matrix, and H is a 4×3 matrix, then $FHGH^T$ is a

A: 2×4	B: 4×2	C: 3×3	D: 4×3	E: 2×3

1 mark 10. If $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \\ 3 & 5 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 4 \\ 0 & -1 & 3 \\ 2 & 7 & 5 \end{bmatrix}$, find the element c_{23} of the matrix $C = AB$.

A: -9	B: 56	C: -3	D: -4	E: 26

1 mark 11. Find the value of x_4 in the solution to the system $A\mathbf{x} = \mathbf{b}$, if $\mathbf{x}$, $\mathbf{b}$ and the inverse of A are as shown below.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \\ 2 \end{bmatrix} \qquad \text{and } A^{-1} = \begin{bmatrix} 2 & 6 & 9 & 1 & 3 \\ 1 & -3 & 5 & 8 & 2 \\ 3 & 7 & 10 & 6 & 0 \\ -2 & 3 & 0 & -5 & -1 \\ 0 & 5 & 3 & 6 & 2 \end{bmatrix}$$

A: 0	B: -3	C: 10	D: 7	E: -7

1 mark 12. A is a 5×7 matrix with rank 4. How many solutions does the system $A\mathbf{x} = \mathbf{0}$ have?

A: No solutions.	B: A 1-parameter family of solutions.
C: A 2-parameter family of solutions.	D: A 3-parameter family of solutions.
E: Cannot be determined.	

1 mark 13. A system of linear equations has 2 equations in 5 unknowns. Which **one** of the following statements is **true**?

A:	The system always has a solution.
B:	The system never has a solution.
C:	If the system has a solution, the family of solutions must have at least 3 parameters.
D:	If the system has a solution, the solution must be unique.
E:	If the system has a solution, the family of solutions must have exactly 3 parameters.

1 mark 14. Let $A\mathbf{x} = \mathbf{b}$ be a system of linear equations. Which **one** of the following statements is **false**?

A:	If the system is homogeneous, then $\mathbf{b} = \mathbf{0}$.
B:	If the system is homogeneous, then it must have only the trivial solution.
C:	If A is invertible, then the system has a unique solution.
D:	If $\text{rank } A = \text{rank } [A \mid \mathbf{b}]$, then the system has at least one solution.
E:	If $\text{rank } A < \text{rank } [A \mid \mathbf{b}]$, then the system has no solution.

1 mark 15. A is an $n \times n$ matrix whose row-reduced echelon form contains exactly **one** row of zeros. Which **one** of the following statements is **always true**?

A:	A is invertible.
B:	The rank of A is n.
C:	The system $A\mathbf{x} = \mathbf{b}$ has no solution.
D:	The system $A\mathbf{x} = \mathbf{0}$ has a unique solution.
E:	The system $A\mathbf{x} = \mathbf{0}$ has a 1-parameter family of solutions.

1 mark 16. Which **one** of the following statements is **true**?

A:	Every homogeneous linear system of 6 equations in 6 unknowns has only the trivial solution.
B:	Every linear system of 6 equations in 6 unknowns has a unique solution.
C:	Every homogeneous linear system of 5 equations in 7 unknowns has infinitely many solutions.
D:	Every homogeneous linear system of 7 equations in 5 unknowns has infinitely many solutions.
E:	Every linear system of 7 equations in 5 unknowns has infinitely many solutions.

1 mark 17. Find $\det \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$.

A: 14	B: 26	C: -14	D: -26	E: 10

1 mark 18. Find det $\begin{bmatrix} 8 & 2 & -16 \\ 1 & 16 & -2 \\ -2 & -4 & 4 \end{bmatrix}$.

A: 16	B: -1	C: -16	D: 1	E: 0

1 mark 19. Find det $\begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -1 \\ 3 & -2 & 0 & 4 \\ 0 & 5 & 1 & -6 \end{bmatrix}$.

A: 0	B: 30	C: -15	D: -30	E: 15

1 mark 20. Find det $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$.

A: 9	B: 6	C: -6	D: -9	E: 15

1 mark 21. Find det $\begin{bmatrix} 1 & 3 & 5 & 7 \\ -2 & -7 & -7 & -7 \\ 3 & 9 & 15 & 14 \\ 2 & 6 & 9 & 14 \end{bmatrix}$.

A: -7	B: 14	C: 0	D: 7	E: -14

1 mark 22. Find det $\begin{bmatrix} a & b & c \\ 2a+b+c & a+2b+c & a+b+2c \\ b+c & a+c & a+b \end{bmatrix}$.

A: 0	B: 1	C: 2	D: abc	E: $a+b+c$

1 mark 23. The 2,1-cofactor of $\begin{bmatrix} 4 & 2 & 4 \\ 3 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is

A: 4	B: -2	C: -4	D: 2	E: -3

Use the following information for questions 24, 25 and 26.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \text{and} \quad \det A = 5$$

1 mark 24. Find $\det \begin{bmatrix} g & d & a \\ h & e & b \\ i & f & c \end{bmatrix}$.

A: -5	B: $-\frac{1}{5}$	C: 5	D: $\frac{1}{5}$	E: -25

1 mark 25. Find $\det \begin{bmatrix} a+d & b+e & c+f \\ 5g & 5h & 5i \\ d & e & f \end{bmatrix}$.

A: -5	B: $-\frac{1}{5}$	C: 5	D: $\frac{1}{5}$	E: -25

1 mark 26. Find $\det \begin{bmatrix} a-2g & b-2h & c-2i \\ a & b & c \\ d & e & f \end{bmatrix}$.

A: 25	B: $-\frac{1}{10}$	C: -10	D: $\frac{1}{10}$	E: 10

1 mark 27. If A is a 3×3 matrix and $\det(2A) = 4$, find $\det A$.

A: 8	B: $\frac{1}{2}$	C: 2	D: $\frac{1}{4}$	E: 32

1 mark 28. If A is a 4×4 matrix and $\det A = \frac{1}{2}$, which **one** of the following statements is **false**?

A: $\det(A^T) = \frac{1}{2}$	B: $\det(2A) = 8$	C: $(\det A)^2 = \det(A^2)$
D: $\det\left(A^{-1}\right) = -\frac{1}{2}$	E: A is invertible.	

1 mark 29. If A is a 5×5 matrix and $\det\left(A^{-1}\right) = \frac{1}{2}$, then $\det(\text{Adj } A)$ is

A: 16	B: 32	C: $\frac{1}{8}$	D: 8	E: $\frac{1}{16}$

1 mark 30. If A and B are both 4×4 matrices such that $\det A = 3$ and $\det B = 5$, find $\det\left(A^{-1}B\right)$.

A: 15	B: $\frac{5}{3}$	C: -15	D: $\frac{3}{5}$	E: $\frac{1}{15}$

1 mark 31. If A and B are square matrices of the same order and A is not invertible, then AB is not invertible. True or False?

A: True	B: False

1 mark 32. For the system of linear equations $\begin{array}{rcrcr} 3x & + & ay & = & 7 \\ -2x & + & by & = & -3 \end{array}$ suppose $\det\begin{bmatrix} 3 & a \\ -2 & b \end{bmatrix} = 10$. Find the value of y in the solution to the system.

A: 1	B: $\frac{1}{5}$	C: $\frac{1}{4}$	D: $\frac{1}{3}$	E: $\frac{1}{2}$

1 mark 33. If $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$, then the $(2,1)$-entry of Adj A is

A: 3	B: 2	C: -2	D: -3	E: 1

1 mark 34. If $A = \begin{bmatrix} a & b & c \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{bmatrix}$ and $\text{Adj } A = \begin{bmatrix} -18 & 18 & 4 \\ 10 & -10 & -2 \\ -11 & 12 & 2 \end{bmatrix}$, find $\det A$.

A: 0	B: 1	C: 2	D: 3	E: 4

1 mark 35. If $A^{-1} = \begin{bmatrix} 1 & a & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & b & 1 \end{bmatrix}$, find $\det(\text{Adj } A)$.

A: $\frac{1}{3}$	B: 3	C: $\frac{1}{9}$	D: 0	E: 9

PART B (15 marks)

SHOW YOUR WORK

2 marks 36. Find an equation in standard form of the plane which passes through the point $(1, -2, 3)$ and is also parallel to the two lines $(x, y, z) = (7, -5, 2) + t(1, -1, 2)$ and $(x, y, z) = (6, 2, -3) + s(2, 1, -1)$. **Show your work.**

Answer: ________________________

2 marks 37. Find the area of the parallelogram determined by $(0, 2, 2)$ and $(1, 0, 2)$. **Show your work.**

Answer: ________________

2 marks 38. Solve the system of linear equations

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & - & x_3 & + & 3x_4 & = & 1 \\ 2x_1 & - & 4x_2 & + & x_3 & & & = & 5 \\ x_1 & - & 2x_2 & + & 2x_3 & - & 3x_4 & = & 4 \end{array}$$

Express your answer in vector form. **Show your work.**

Answer: ______________________________

2 marks 39. Find the rank of the given matrix. **Show your work.**

$$\begin{bmatrix} 1 & -1 & 2 & 3 & 0 \\ -1 & 0 & -4 & 3 & -1 \\ 2 & -1 & 6 & 0 & 1 \\ -1 & 2 & 0 & -1 & 1 \end{bmatrix}$$

Answer: ______________________

2 marks 40. Use cofactor expansion along column 2 to find the determinant of the following matrix. **Show your work.**

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & -4 \\ 0 & 2 & 3 \end{bmatrix}$$

Answer: ____________________

2 marks 41. Find the determinant of

$$\begin{bmatrix} 2 & 4 & 0 \\ 6 & 11 & 3 \\ 4 & 10 & 5 \end{bmatrix}$$

using elementary row operations to transform the matrix to upper triangular form. **Show your work.**

Answer: ____________________

3 marks 42. Use Cramer's rule to find the value of y in the solution to the system of linear equations shown below. **Show your work.**

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & -1 \\ 2x & + & y & + & 3z & = & 0 \\ 3x & - & 3y & + & 2z & = & 1 \end{array}$$

Answer: ______________________

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Final Examination

Thursday, December 16, 2010 **Code 111** 9:00 a.m. - 12:00 noon

INSTRUCTIONS

1. DO NOT UNSTAPLE THE BOOKLET. The three blank pages at the back of the booklet may be torn off and used for rough work. Do not tear any other pages out of the booklet.
2. CALCULATORS AND NOTES ARE NOT PERMITTED.
3. There are two parts to this examination: PART A (35 marks) in multiple choice format and PART B (15 marks) in show your work format.
4. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil.
5. In Part B, show all your work in the space provided.
6. Questions are printed on both sides of the paper, they begin on Page 1 and continue to Page 11. Be sure that your booklet is complete.
7. Fill in the top of this page, **and the next page**, completely.
8. Fill in the top of the scantron card with your Name and Student Number (**both printing and coding**) and also your class section and the exam CODE indicated above.
9. You must hand in this question paper, your scantron card, and all rough work sheets.
10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Sinnamon	Main	9:30 MWF	001
Lowrey	Main	12:30 MWF	002
Olds	Main	1:30 MWTh	003
Florence	Brescia	10:30 MWTh	530
Kuzmin	Huron	12:30 MWF	550
Kuzmin	Huron	8:30 MWF	551
Meredith	King's	1:30 TTh	570
Meredith	King's	9:30 TTh	571

11. TOTAL MARKS = 50.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
1–6	
7	
8	
9	
10	
11	
TOTAL	