PART A (35 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

1 mark 1. If $\mathbf{u} = (1, -1, 3)$ and $\mathbf{v} = (2, 1, -1)$. Find $\mathbf{u} \bullet \mathbf{v}$.

A: -2	B: -1	C: 0	D: 1	E: 2

1 mark 2. If $\mathbf{u} = (1, -1, 3)$ and $\mathbf{v} = (2, 1, -1)$, find $\mathbf{u} \times \mathbf{v}$.

A: $(2, -1, -3)$	B: $(-2, -7, 3)$	C: $(-2, 7, 3)$	D: $(-3, 7, 3)$	E: $(-3, 5, 4)$

1 mark 3. Which one of the following is an equation of the plane through the point $(1, 4, -5)$ with normal $(1, 3, 2)$?

A: $x + 12y - 10z = 0$	B: $x + 3y + 2z = 0$	C: $x + 4y - 5z = 0$
D: $x + 4y - 5z = 3$	E: $x + 3y + 2z = 3$	

1 mark 4. Find a vector parallel to the line $6x - 5y = 3$.

A: $(6, 5)$	B: $(5, 6)$	C: $(-5, 6)$	D: $(6, -5)$	E: $(3, 3)$

1 mark 5. Which one of the following describes the line through the points $(2, 1)$ and $(3, 4)$?

A: $x + 3y = 5$	B: $3x - y = 2$	C: $(x, y) = (1, -2) + t(2, 6)$
D: $(x, y) = (2, 1) + t(3, 1)$	E: $(x, y) = (3, 4) + t(3, -1)$	

1 mark 6. Find $\cos\theta$ where θ is the angle between vectors $\mathbf{u} = (2, 0, 4, 1, -2)$ and $\mathbf{v} = (1, 1, -1, -1, 0)$.

A: $-\frac{3}{10}$	B: $\frac{3}{10}$	C: $-\frac{3}{7}$	D: $\frac{3}{7}$	E: $\frac{7}{10}$

1 mark 7. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ -2 & -6 & 5 \\ 3 & 9 & -3 \end{bmatrix}$ and let B be the row-reduced echelon form of A. Find the first row of matrix B.

A: $[1\ \ 0\ \ 0]$	B: $[1\ \ 3\ \ -1]$	C: $[1\ \ 3\ \ 2]$	D: $[1\ \ 3\ \ 0]$	E: $[1\ \ 0\ \ 3]$

1 mark 8. If $\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$ is the augmented matrix for a particular system of linear equations, how many solutions does the system have?

A: no solution	B: exactly one solution
C: a one-parameter family of solutions	D: a two-parameter family of solutions
E: a three-parameter family of solutions	

1 mark 9. Find the value(s) of k for which the system of linear equations with augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & k \\ 0 & 2 & k & 2 \end{array}\right]$ has no solution.

A: $k = 1$ only	B: all $k \neq 1$	C: $k = 4$ only	D: all $k \neq 4$	E: no value of k

1 mark 10. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ and $C = (A+B)^T = [c_{ij}]$. Determine the value of c_{12}.

A: 0	B: 1	C: 2	D: 3	E: 4

1 mark 11. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, I is the 2×2 identity matrix and $A^2 + kA = I$, find the value of k.

A: 0	B: -1	C: 1	D: -2	E: 2

1 mark 12. If $A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, where $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, find the value of y in the solution to the system of linear equations

$$\begin{array}{rcrcrcl} ax & + & by & + & cz & = & 3 \\ dx & + & ey & + & fz & = & 2 \\ gx & + & hy & + & iz & = & 1 \end{array}$$

A: 2	B: 3	C: 4	D: 5	E: 7

1 mark 13. If A is a square matrix such that $A^3 = I$, which one of the following must always be true?

A: $A = I$	B: $A^2 = I$	C: $A^T = A$	D: $A^{-1} = A^2$	E: A is a zero matrix

1 mark 14. Which one of the following **must have** infinitely many solutions?

A: A system of linear equations with 7 equations in 11 unknowns.
B: A system of linear equations with 11 equations in 7 unknowns.
C: A homogeneous system of linear equations with 11 equations in 7 unknowns.
D: A homogeneous system of linear equations with 7 equations in 11 unknowns.
E: A homogeneous system of linear equations with 7 equations in 7 unknowns.

1 mark 15. If the rank $A = 4$ and rank $[A \mid \mathbf{b}] = 5$, which one of the following describes the solution to the system of linear equations $A\mathbf{x} = \mathbf{b}$?

A: no solution	B: exactly one solution
C: exactly one parameter in the solution	D: exactly four parameters in the solution
E: exactly 5 parameters in the solution	

1 mark 16. If A is an $n \times n$ invertible matrix, which of the following statements are **false**?

(i) The rank of A is n.
(ii) $A\mathbf{x} = \mathbf{b}$ has a unique solution for any $\mathbf{b}$.
(iii) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
(iv) The row-reduced echelon form of A is an identity matrix.

A: (i) only	B: (ii) and (iii) only	C: (iv) only
D: (i) and (iv) only	E: none of (i), (ii), (iii) or (iv)	

1 mark 17. The coefficient matrix of a homogeneous system of 10 linear equations in 15 unknowns has rank 4. How many parameters are there in the solution to the system?

A: 4	B: 5	C: 6	D: 10	E: 11

1 mark 18. Consider the linear system $A\mathbf{x} = \mathbf{b}$ where A is a 5×5 invertible matrix. Which one of the following statements is **false**?

A: There is a row of five zeroes in the row-reduced echelon form of A.
B: The rank of A is 5.
C: The rank of $[A \mid \mathbf{b}]$ is 5.
D: The system $A\mathbf{x} = \mathbf{b}$ has a unique solution.
E: The system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

1 mark 19. Find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$.

A: 0	B: 1	C: 2	D: 3	E: 4

1 mark 20. What is the 2,3-cofactor of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 1 & -1 & 4 \end{bmatrix}$?

A: 1	B: -1	C: 5	D: -5	E: 6

1 mark 21. If $\det \begin{bmatrix} 2 & 3 \\ 4 & k \end{bmatrix} = 10$, what is the value of k?

A: 6	B: -1	C: 12	D: 20	E: 11

1 mark 22. Which of the following have the value 8?

(i) $\det \begin{bmatrix} 2 & 3 & 6 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \end{bmatrix}$ (ii) $\det \begin{bmatrix} 0 & 0 & 4 \\ 0 & 1 & 5 \\ 2 & 3 & 6 \end{bmatrix}$ (iii) $\det \begin{bmatrix} 4 & 0 & 0 \\ 5 & 1 & 0 \\ 6 & 3 & 2 \end{bmatrix}$

A: (i) and (ii) only	B: (i) and (iii) only	C: (ii) and (iii) only
D: all of them	E: none of them	

1 mark 23. Find $\det \begin{bmatrix} -1 & 1 & -3 & -2 \\ 3 & 2 & 2 & 6 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 4 \end{bmatrix}$.

A: 0	B: 4	C: 12	D: 2	E: 6

Use the following information for questions 24, 25 and 26.

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 3$$

1 mark 24. Find $\det \begin{bmatrix} d & a & g \\ e & b & h \\ f & c & i \end{bmatrix}$.

A: 3	B: $\frac{1}{3}$	C: 0	D: $-\frac{1}{3}$	E: -3

1 mark 25. Find $\det \begin{bmatrix} 2a & d & d+g \\ 2b & e & e+h \\ 2c & f & f+i \end{bmatrix}$.

A: $-\frac{3}{2}$	B: 6	C: $\frac{3}{2}$	D: -6	E: 3

1 mark 26. Find $\det \begin{bmatrix} d & e & f \\ 3a & 3b & 3c \\ 2a-g & 2b-h & 2c-i \end{bmatrix}$.

A: 9	B: -9	C: 6	D: -18	E: 18

1 mark 27. A and B are 3×3 matrices with $\det A = 5$ and $\det B = 2$. Find $\det\left(A^{-1}B^T\right)$.

A: $\frac{5}{2}$	B: $\frac{2}{5}$	C: $-\frac{5}{2}$	D: $-\frac{2}{5}$	E: None of A, B, C or D.

1 mark 28. A and B are 4×4 matrices with $\det A = 5$ and $\det B = 2$. Which of the following are **true**?

(i) $\det(-A) = -5$
(ii) $\det(3B) = 81$

A: (i) only	B: (ii) only	C: both (i) and (ii)	D: neither (i) nor (ii)

1 mark 29. For $n \times n$ matrices A and B, which of the following must **always** be true?

(i) $\det(A+B) = \det A + \det B$
(ii) $\det(AB) = (\det A)(\det B)$
(iii) If $\det A = \det B$ then $A = B$

A: all of them	B: none of them	C: (ii) and (iii) only
D: (ii) only	E: (iii) only	

1 mark 30. Let A be an invertible $n \times n$ matrix. Which one of the following is **false**?

A: If $A^TA = I$ then $\det A = \pm 1$.	B: If $A = A^{-1}$ then $\det A = \pm 1$.
C: If $A^T = A^{-1}$ then $\det A = \pm 1$.	D: If $A^T = A$ then $\det A = \pm 1$.
E: If $A^2 = I$ then $\det A = \pm 1$.	

1 mark 31. If A is a 4×4 matrix and $\det A = 3$, what is the rank of A?

A: 0	B: 1	C: 2	D: 3	E: 4

1 mark 32. For the system of linear equations

$$\begin{array}{rcrcl} 3x & + & ay & = & 4 \\ -2x & + & by & = & 2 \end{array}$$

suppose that $\det \begin{bmatrix} 3 & a \\ -2 & b \end{bmatrix} = -7$. Find the value of y in the solution to the system.

A: $\frac{1}{2}$	B: $-\frac{1}{2}$	C: 2	D: -2	E: 0

1 mark 33. If A is a 4×4 matrix with $\det A = 3$, find $\det(\text{Adj } A)$.

A: 3	B: 9	C: 12	D: 27	E: 81

1 mark 34. A is a 3×3 matrix with $\det A = -3$ and $\text{Adj } A = \begin{bmatrix} 3 & 1 & 4 \\ -6 & -3 & -6 \\ 0 & -1 & -1 \end{bmatrix}$.

Find the $(2,3)$-entry of A^{-1}.

A: 2	B: -6	C: -1	D: $\frac{1}{3}$	E: 3

1 mark 35. If $A = \begin{bmatrix} a & b & c \\ 5 & 1 & 5 \\ d & e & f \end{bmatrix}$ and $\text{Adj } A = \begin{bmatrix} 6 & 4 & 10 \\ -30 & -24 & -30 \\ 0 & -4 & -4 \end{bmatrix}$, what is the value of $\det A$?

A: 6	B: -24	C: -4	D: -84	E: -324

PART B (15 marks)

SHOW YOUR WORK

2 marks 36. Find the area of the parallelogram determined by the vectors $\mathbf{u} = (1, 0, 1)$ and $\mathbf{v} = (0, 1, 1)$. **Show your work.**

2 marks 37. Find the point of intersection of the line $(x, y, z) = (2, -1, 4) + t(1, 2, -1)$ with the plane $3x + y - z = 31$. **Show your work.**

2 marks 38. Use Gauss-Jordan Elimination to solve the system of linear equations

$$\begin{aligned} x &+ y &+ 2z &= 1 \\ 2x &+ 2y &+ 4z &= 2 \\ -x &- y &- 2z &= -1 \end{aligned}$$

Express your answer in vector form. **Show your work.**

3 marks 39. Find the determinant of

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ -2 & 1 & 2 & -3 \\ 0 & 0 & 3 & 1 \\ 1 & 0 & 0 & 4 \end{bmatrix}$$

using elementary row operations to transform the matrix to upper triangular form. **Show your work.**

4 marks 40. Use Cramer's rule to find the value of z in the solution to the system of linear equations shown below. **Show your work.**

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + z &= 1 \\ -x + y + 2z &= 6 \end{aligned}$$

2 marks 41. Find the adjoint of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A Final Examination

Saturday, December 10, 2011 **Code 111** 2:00 p.m. - 5:00 p.m.

INSTRUCTIONS

1. Fill in the top of this page, **and the next page**, completely.
2. Fill in the top of the scantron card with your Name and Student Number (**both printing and coding**) and also your class section and the exam **Code** indicated above.
3. DO NOT UNSTAPLE THE BOOKLET.
4. CALCULATORS AND NOTES ARE NOT PERMITTED.
5. There are two parts to this examination: PART A (35 marks) in multiple choice format and PART B (15 marks) in show your work format.
6. In Part A, **circle** the correct answer to each question **on this paper** AND fill in the appropriate box on the **scantron** card with an HB pencil.
7. In Part B, show all your work in the space provided.
8. Questions are printed on both sides of the paper, they begin on Page 1 and continue to Page 9. Be sure that your booklet is complete.
9. You must hand in this question paper, your scantron card, and all rough work sheets.
10. Circle your section in the list below.

Instructor	Campus/College	Time	Section
Olds	Main	9:30 MWF	001
Tohaneanu	Main	12:30 MWF	002
Tohaneanu	Main	1:30 MWTh	003
Garrousian	Main	7:00 Tu	004
Florence	Brescia	1:30 MWTh	530
Pourkia	Huron	11:30 TuThF	550
Pourkia	Huron	10:30 TuThF	551
Meredith	King's	1:30 TTh	570
Meredith	King's	9:30 TTh	571

11. TOTAL MARKS = 50.

Student Number (**Print**)

Student's Name (**Print**)

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PAGE	MARK
1–5	
6	
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TOTAL	