CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 6651: Algorithm Design Techniques

Fall 2015

Quiz # 1

First Name	**Last Name**	**ID#**

Question 1

Solve the following recurrence equation

$$\begin{aligned} t_n &= 2t_{n-2} - t_{n-4} && n \geq 4 \\ t_n &= n && 0 \leq n \leq 3. \end{aligned}$$

Express your solution with the simplest expression using the Θ notation.

Characteristic equation : $x^4 - 2x^2 + 1 = 0$ **(2 points)** $\rightsquigarrow (x^2-1)^2$ because of the identity:

$$(a-b)^2 = a^2 + b^2 - 2ab.$$

Then $x^2 - 1$ is a difference of two squared terms, we use the identity

$$a^2 - b^2 = (a-b)(a+b).$$

It leads to: $x^2 - 1 = (x-1)(x+1)$.
Characteristic equation is then equivalent to:

$$(x-1)^2(x+1)^2 = 0. \qquad \textbf{(2 points)}$$

Both two roots 1 and -1 are of multiplicity two. **(2 points)**

$$G(n) = (C_1 + C_2 n)(1)^n + (C_3 + C_4 n)(-1)^n = C_1 + C_2 n + (C_3 + C_4 n)(-1)^n \quad \textbf{(2 points)}$$

$n = 2k$**:** $G(2k) = C_1 + C_3 + n(C_2 + C_4) = A_1 + B_1 n$

$n = 2k+1$**:** $G(2k+1) = C_1 - C_3 + n(C_2 - C_4) = A_2 + B_2 n$

It follows:

$$C_1 = \frac{A_1 + A_2}{2}, \quad C_2 = \frac{B_1 + B_2}{2}, \quad C_3 = \frac{A_1 - A_2}{2}, \quad C_4 = \frac{B_1 - B_2}{2}.$$

Clearly, the coefficient of n is not zero, as otherwise t_n = constant and then the recurrence solution is not satisfied unless $t_n = 0$, consequently

$$t_n = \Theta(n). \qquad \textbf{(2 points)}$$

Question 2

Give the mathematical definition of $\Omega(n)$ notation. Using this definition, show that: $n^2 + 100 = \Omega(\log n)$.

Mathematical definition of Ω-notation: $f(n) = \Omega(g(n))$ if there exists positive constants c, n_0 such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0$. **(5 points)**

See Figure 1 for the graph of the log function.

For $n \geq n_0 = 1$, we have:

$$n^2 + 100 \geq n^2 \geq n \geq \log n.$$

Therefore, definition applies with $c = 1$ and $n_0 = 1$. **(5 points)**

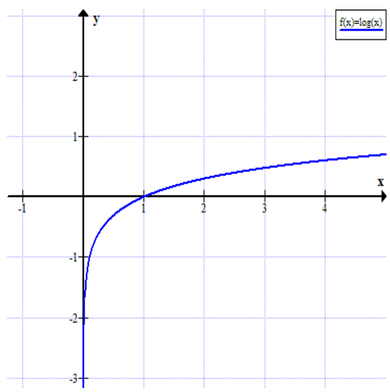

Figure 1: Graph of log function