Q1. Let A, H, J, P, R, S, U denote the following propositional variables: [6 points]

A : "The shark attacks Pat"
H : "The shark is hungry."
J : "Jo is a knight."
P : "Pat is a knight."
R : "Jo rescues Pat."
S : "Pat goes swimming around the island "
U : "Swimming around the island is unsafe."

Translate each of the following sentences into compound propositions using the above propositional variables, logical connectives, and parentheses when appropriate. *Parentheses have been added to some sentences for clarification; otherwise, follow the conventions of precedence of logical connectives.* Assume that Jo and Pat are inhabitants of the Island of Knights & Knaves.

(a) The shark attacks Pat only if the shark is hungry and Pat is a knave.

(b) A necessary condition for Jo to rescue Pat is that $\Big($Pat goes swimming around the island whenever swimming around the island is unsafe$\Big)$.

(c) A sufficient condition for swimming around the island to be safe is that the shark is not hungry.

(d) Either Jo is a knight or the shark does not attack Pat.

(e) (If the shark attacks Pat, then Jo rescues Pat) unless Pat is a knave.

(f) Whenever the shark is hungry, Pat goes swimming around the island, but not conversely.

(BONUS) [+2 bonus points] Suppose that all six of the propositions (a)–(f) are **true**. What (if anything) can you conclude? Circle the best answer for each question. You do not need to show any justification for this, but you will only earn the bonus points if all of your answers are correct.

Does the shark attack Pat?	*Circle:*	**YES**	**NO**	unable to determine
Is the shark hungry?	*Circle:*	**YES**	**NO**	unable to determine
Is Jo a knight?	*Circle:*	**YES**	**NO**	unable to determine
Is Pat a knight?	*Circle:*	**YES**	**NO**	unable to determine
Does Pat go swimming around the island?	*Circle:*	**YES**	**NO**	unable to determine

Q2.

[6 points]

Here is a truth table for three *mystery* compound propositions P_1, P_2, and P_3, each consisting of the propositional variables w, x, y, z.

w	x	y	z	P_1	P_2	P_3
F	**F**	**F**	**F**	**T**	**F**	**T**
F	**F**	**F**	**T**	**F**	**F**	**T**
F	**F**	**T**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**	**F**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**	**F**
F	**T**	**T**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**F**	**F**	**F**
T	**F**	**F**	**T**	**T**	**F**	**F**
T	**F**	**T**	**F**	**F**	**T**	**F**
T	**F**	**T**	**T**	**T**	**F**	**T**
T	**T**	**F**	**F**	**T**	**F**	**F**
T	**T**	**F**	**T**	**T**	**F**	**T**
T	**T**	**T**	**F**	**T**	**T**	**F**
T	**T**	**T**	**T**	**T**	**F**	**F**

(2 pts) **a.** Is $(P_1 \wedge P_2) \rightarrow P_3$ a tautology? If so, explain. If not, give all counterexamples.

(2 pts) **b.** Is $\neg\Big(P_3 \rightarrow (P_1 \vee P_2)\Big)$ a contradiction? If so, explain. If not, give all counterexamples.

(2 pts) **c.** Give a DNF for P_3

Q3. Using the Laws from the Table of Logical Equivalences, prove that [5 points]

$$\neg\Big[\big(\neg P \to (Q \wedge R)\big) \to \neg(R \to P)\Big] \equiv P$$

You may use **one or two** laws at each step, and you must write the name of the law(s) used at each step. Do not combine more than two laws in one step.