MATH 317: Numerical Analysis

Mid-term test - 20 Oct., 2010 (11:35 - 12:25 pm)

Answer all questions. They have equal value.

1. Answer briefly 3 of the following 4 questions:

a) Rewrite the binary number 11011 in base 10 digits, i.e. human.

b) Say we have three data points (x_0, f_0), (x_1, f_1) and (x_2, f_2). Use Lagrange interpolation and write down the interpolating polynomial.

c) Why is numerical integration "safer" than numerical differentiation?

d) How does Gaussian quadrature improve upon the accuracy of merely fitting polynomials to uniformly-spaced data points?

2. Say we have a computer that can do everything except calculate cube roots. Show how you could use the Newton-Raphson method on $f(x) = x^3 - a$ to calculate $a^{1/3}$. Will it converge? Either way, prove your answer.

3. Let's say we want to improve upon Simpson's rule for integration by fitting a cubic polynomial to the points (x_j, f_j) for $j = 0, 1, 2, 3$, where $x_o = a$ and $x_3 = b$. The goal is to approximate $\int_a^b f(x)\, dx$ by $\sum_{i=0}^{3} C_i f(x_i)$. Show how you would go about finding the C_i. Note: the algebra gets a little messy. Just explain how you would set up the problem for solution.