ENGR 301 Engineering Management Principles and Economics

Tutorial 8: Economic Equivalency

1. An individual deposits an annual bonus into a savings account that pays 8% interest compounded annually. The size of the bonus increases by $200 each year, and the initial bonus amount was $5000. Determine how much will be in the account immediately after the 5th deposit.

   To determine how much is available in the account after the 5th deposit we first determine a single present amount at year 0.

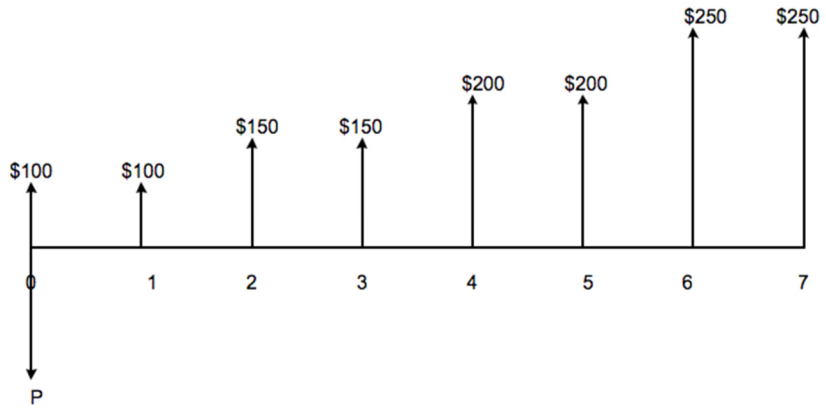
   $5000(P/A,8%,5) + $200(P/G,8%,5) = $5000(3.993)+$200(7.372) = $21,439
   or
   $5000(P/A,8%,5) + $200(P/A,8%,5)(A/G,8%,5) = $5000(3.993)+$200(3.993)(1.846) = $21,439

   Convert this present amount to a future amount at t=5
   $21,439(F/P,8%,5) = $21,439(1.469) = $31,494

   Or calculate the future amount at t=5 directly:

   F = $5000(F/A,8%,5) + $200(F/G,8%,5)= $5000(F/A,8%,5) + $200(F/P,8%,5)(P/G,8%,5)
   F = $5000(5.867) + $200(1.469)(7.372) = $31,501

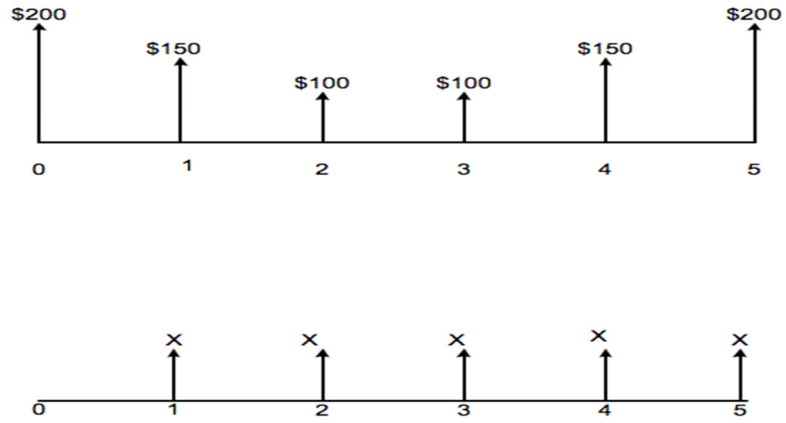
2. Compute the value of P in the accompanying cash flow diagram, assuming that i=9%.



   P = $100 + $100(P/F,9%,1) + $150(P/F,9%,2) + $150(P/F,9%,3) + $200(P/F,9%,4) + $200(P/F,9%,5) + $250(P/F,9%,6) +$ 250(P/F,9%,7)

   P = $991.31

3. The two cash flow transactions shown in the accompanying cash flow diagram are said to be equivalent at 6% interest compounded annually. Find the unknown value of X that satisfies the equivalence.



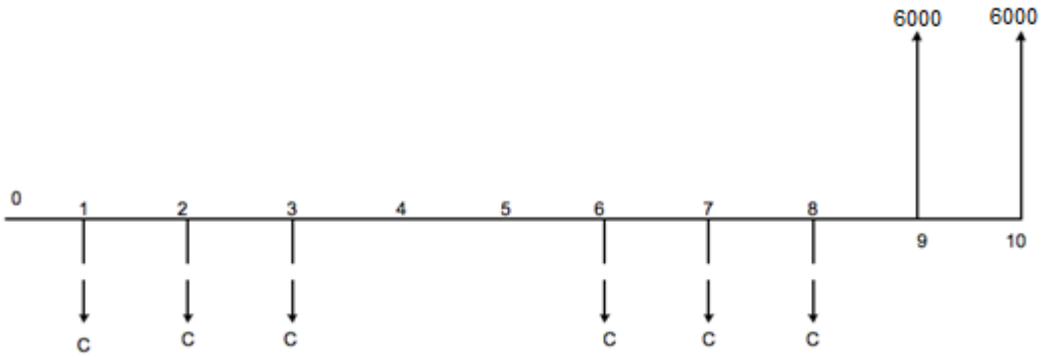
To solve for X, convert the cash flow series to a present amount, then convert to a uniform payment series.

P = $200 + $150(P/F,6%,1) + $100(P/F,6%,2) + $100 (P/F,6%,3) + $150 (P/F,6%,4) + $200(P/F,6%,5)

P = $782.7

X = $782.7(A/P,6%,5) = $782.7(0.2374) = $185.80

4. From the accompanying cash flow diagram, find the value of C that will establish the economic equivalence between the deposit series and the withdrawal series at an interest rate of 8% compounded annually.



Solve by bringing everything to the present and equating.

C(P/A,8%,3) + C(P/A,8%,3)(P/F,8%,5) - $6000(P/A,8%,2)(P/F,8%,8)
C(2.577) + C(2.577(0.6806) - $6000(1.783)(0.5403)
4.3309C - $5780 = 0
C = $1334

5. John Jay is purchasing a $24,000 automobile, which is to be paid for in 48 monthly installments of $543.35. What effective annual interest is he paying for his financing arrangement?

P=$24,000; A=$543.35; n=48; i=?

$24,000 = $543.35 (P/A,i%,48)
(P/A,i%,48) = 44.17

Try different values for i: For i = 1%; (P/A,0.01,48) = 37.97
For i = 0.5%; (P/A,0.005,48) = 42.58
For i= 0.25%; (P/A,0.0025,48) = 45.179

Therefore the monthly interest rate lies between 0.25% and 0.5%

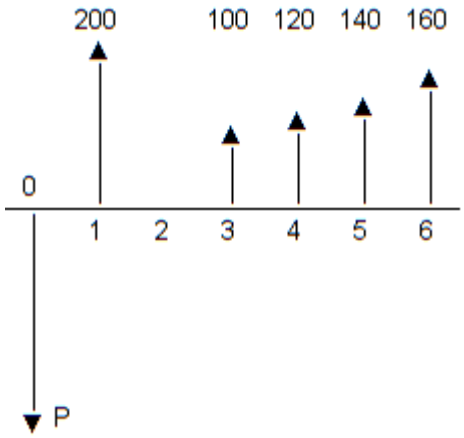
Interpolating: i = 0.25% + (0.5%-0.25%)[(45.179-44.17)/(45.179-42.58)] = 0.0034 = 0.34%

$i_{eff} = (1 + 0.0034)^{12} - 1 = 0.0416 = 4.16\%$

6. Sketch the cash flow diagram associated with the following equivalency expression:
P = $200(P/F,i%,1) + [$100 + $20(A/G,i%,4)](P/A,i%,4)(P/F,i%,2)

We have a gradient series for 4 periods with a step of $20, which we convert into 4 equal amounts (the 20(A/G,1%,4)).
Coinciding with our gradient series we have 4 equal amounts of $100.
We now convert the uniform and gradient series into a single present value one period before the beginning of those series (the (P/A),1%,4)). The (P/F,1%,2) is telling us that this single present value is considered a future value when compared with P, and that this future value is 2 periods after P.
The $200(P/F,i%,1) is telling us we have a single amount of $200 1 period after P.

That gives the following CFD:



P = 200(P/F,0.1,1) + [100+20(A/G,0.1,4)](P/A,i%,4)(P/F,0.1,2)
P = 200(0.90909) + [100+20(1.3812)](3.1699)(0.82645) = 516.16

P = 200(P/F,0.1,1) + 100(P/F,0.1,3) + 120(P/F,0.1,4) + 140(P/F,0.1,5) + 160(P/F,0.1,6)
P = 200(0.90909) + 100(0.75131) + 120(0.68301) + 140(0.62092) + 160(0.56447) = 516.15