

Date: - 14/01/19.

# Variability:-

Range = max. - min.

Variance :- (How far you deviate from mean)

(Square to eliminate -ve values)

$$\sum_{i=1}^n (x_i - \mu)^2$$

$n$   
of a population

$\sigma^2 = \text{population variance}$

Find Sample Variance,

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$n-1$

$\rightarrow -1$  to make  
it a better  
estimate as  
we are finding  
variance of  
sample not population

$$\frac{1}{n-1} \left[ \sum_{i=1}^n x_i^2 - \frac{\left( \sum_{i=1}^n x_i \right)^2}{n} \right]$$

Standard deviation =  $\sigma$  (square root of variance)

Ex-2

A) Range = max - min  
= 65 - 40  
= 25

$$S^2 = \frac{1}{n-1} \left[ \sum_{i=1}^n x_i^2 - \frac{\left( \sum_{i=1}^n x_i \right)^2}{n} \right]$$
$$= \frac{1}{4} \left[ (40^2 + 50^2 + 50^2 + 55^2 + 65^2) - \frac{(260)^2}{5} \right]$$

$$\frac{1}{4} \left[ 13850 - \frac{(260)^2}{5} \right]$$

$$= 82.5$$
$$S = \sqrt{82.5}$$
$$= 9.08$$

if each value +10  
→  $\bar{x}$  is also +10.  
→ variance and standard deviation stays the same.

- if each value  $\times 10$
- $\rightarrow \bar{x}$  is also  $\times 10$
- $\rightarrow$  variance is  $\times 10^2$
- $\rightarrow$  standard deviation  $\times \sqrt{10^2} = \times 10$

- $\therefore$  if each value  $\times 100$
- $\rightarrow \bar{x}$  is also  $\times 100$
- $\rightarrow$  variance is  $\times 100^2$
- $\rightarrow$  S.d is  $\times \sqrt{100^2}$ .

Significance of the Standard deviation:-

$\bar{x}, \sigma$

$(\bar{x} - \sigma, \bar{x} + \sigma)$   $\begin{matrix} \text{upper} \\ \text{limit} \end{matrix}$  at least  $1 - \frac{1}{k^2}$

$\rightarrow$  Lower limit

Let's say  $\bar{x}$  is 60,  $k=2, \sigma=10$

$(\bar{x} - k\sigma, \bar{x} + k\sigma)$   $\rightarrow$  It is a factor we multiply for data coverage.

$(60 - 2(10), 60 + 2(10))$

$= (40, 80)$

$= 1 - \frac{1}{2^2} = 0.75$

$= 75\%$  of the data changes between 40 & 80.

P-3.

$$\bar{x} = 35 \quad n = 5$$

a)  $35 - k(5), 35 + k(5)$

$$35 - 5k = 25$$

$$35 - 25 = 5k$$

$$= \frac{10}{5} = 2.$$

$$35 + 5k = 45$$

$$5k = 10$$

$$k = 2$$

$$1 - \frac{1}{2^2} = 0.75$$

$$= 75\% \text{ (at least)}$$

$$1 - 0.75$$

⊙

⊙

$$35 - 5k = 20$$

$$15 = 5k$$

$$k = 3$$

$$\begin{array}{r} 35 \\ 20 \\ \hline 15 \end{array}$$

⊙

$$1 - \frac{1}{3^2} = \frac{4}{9} = 0.66.$$

$$\frac{4}{9}$$

$$1 - 0.66 = 0.334$$

$$35 + 5k =$$

$$35 + 5k = 50$$

$$k = 3 = 0.66$$

$$= \text{at most } 11.11\%$$

# Ex-4 (Empirical Rule)

a)  $\bar{x} = 40$        $n = 5000$   
 $\sigma = 15$

~~$40 - 15k = 27$~~   
 ~~$15k = 13$~~   
 ~~$k = 1$~~

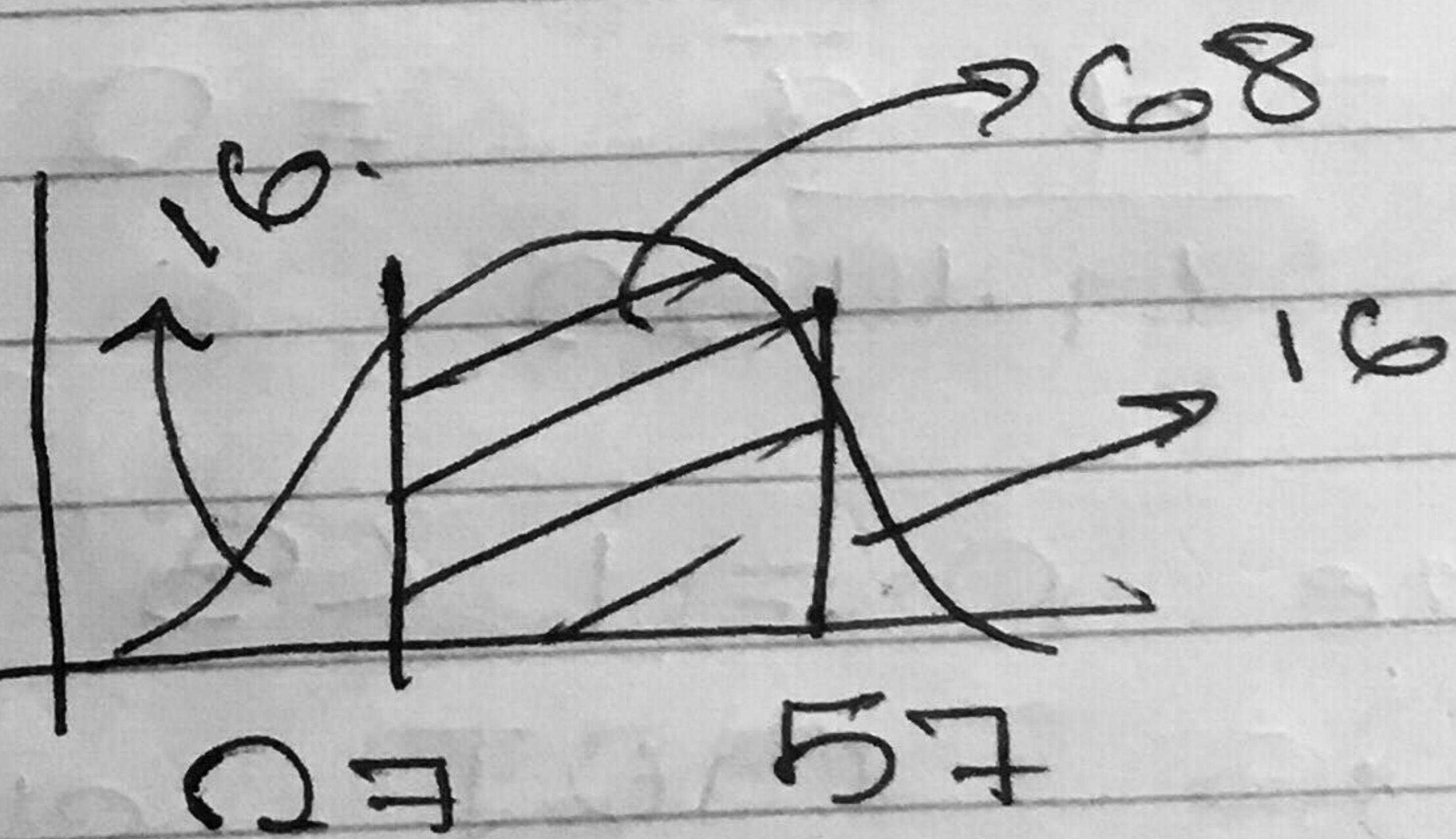
$(27, 57)$   
 $k = \frac{40 - 27}{15}$   
 $= 1$

$k \rightarrow$  is the 'z-score'  
 $k \rightarrow$  how many standard deviations it's away from mean.

distribution is symmetric, so use empirical rule:  
 $\therefore$  Approximately = 68%

b)  $\frac{1 - 0.68}{2}$

= 0.16 (at most)



$k = 1 = 68\%$

$k = 2 = 95\%$

$k = 3 = 99.7\%$

$$Pa = 5$$

$$B = 49$$

$$S = \frac{B}{4} = \frac{\text{max-min}}{4}$$

$$= \frac{31.9 - 10.2}{4}$$

$$= 5.425 \text{ (approximately)}$$

Pa = 6. (How to find outliers)

$$z = \frac{30}{10} = 3$$

$$S = \sqrt{\frac{1}{9} (266 - 90)} = 4.422$$

$$z = \frac{x - \bar{x}}{4.422} = 2.71$$

Since  $2.71 < 3$ , the value  
15 is NOT an outlier.

