[1] **1.** Find the domain of the function $g(x) = \dfrac{e^{\sqrt{1-2x}}}{x^2 - 9}$.

solution: We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have $1 - 2x \geq 0$ which gives $x \leq 1/2$ and we also need to have $x^2 - 9 \neq 0$ which gives $x \neq \pm 3$. So the domain is $(-\infty, -3) \cup (-3, 1/2]$.

[2] **2.** Using the definition of the derivative, find $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{9 - 2x}$.

solution:

$$
\begin{aligned}
&\lim_{h\to 0} \frac{\sqrt{9-2(x+h)} - \sqrt{9-2x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{9-2(x+h)} - \sqrt{9-2x}}{h} \times \frac{\sqrt{9-2(x+h)} + \sqrt{9-2x}}{\sqrt{9-2(x+h)} + \sqrt{9-2x}} \\
&= \lim_{h\to 0} \frac{\left(\sqrt{9-2(x+h)}\right)^2 - \left(\sqrt{9-2x}\right)^2}{h\left(\sqrt{9-2(x+h)} + \sqrt{9-2x}\right)} \\
&= \lim_{h\to 0} \frac{(9-2(x+h)) - (9-2x)}{h\left(\sqrt{9-2(x+h)} + \sqrt{9-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2h}{h\left(\sqrt{9-2(x+h)} + \sqrt{9-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2}{\left(\sqrt{9-2(x+h)} + \sqrt{9-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2}{\left(\sqrt{9-2(x+0)} + \sqrt{9-2x}\right)} \\
&= \lim_{h\to 0} \frac{-2}{(2\sqrt{9-2x})} \\
&= \lim_{h\to 0} \frac{-1}{(\sqrt{9-2x})}
\end{aligned}
$$

[4] **3.** Evaluate each of the limits. Show your work!

a) $\displaystyle\lim_{x\to 2} \frac{x-2}{2x^2 - 4x}$

solution:

$$\begin{aligned}\lim_{x\to 2}\frac{x-2}{2x^2-4x} &= \lim_{x\to 2}\frac{x-2}{2x(x-2)}\\ &= \lim_{x\to 2}\frac{1}{2x}\\ &= \frac{1}{2(2)}\\ &= \frac{1}{4}\end{aligned}$$

b) $\displaystyle\lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}}$

solution:

$$\begin{aligned}\lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}} &= \lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}}\times\frac{1/x^3}{1/x^3}\\ &= \lim_{x\to\infty}\frac{1-1/x^2-7/x^3}{2+1/x^{5/2}}\\ &= \lim_{x\to\infty}\frac{1-0-0}{2+0}\\ &= \frac{1}{2}\end{aligned}$$

[**2**] **4.** Suppose that $x^3+x^2y-2y^5=5$. Find $y'=\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

solution:

$$\begin{aligned}x^3+x^2y-2y^5 &= 5\\ 3x^2+2xy+x^2y'-10yy' &= 0\\ x^2y'-10yy' &= -3x^2-2xy\\ y' &= \frac{-3x^2-2xy}{x^2-10y}\end{aligned}$$

[**2**] **5.** Let $f(x)=2x^3-x$. Find the equation of the line tangent to f at the point $x=1$.

solution: First we find the derivative: $f'(x)=6x^2-1$. The slope is $f'(1)=5$. The value of the function is $f(1)=1$. Then we have the line.

$$\begin{aligned}(y-y_0) &= m(x-x_0)\\ (y-1) &= 5(x-1)\\ y &= 5x-4\end{aligned}$$

[6] **6.** Differentiate each of the following. Show your work! You do not need to simplify your answer.

a) $u(t) = (t^2+5)^7(e^t+\pi)$

solution:

$$7(t^2+5)^6(2t)(e^t+\pi)+(t^2+5)^7(e^t)$$

b) $g(x) = \dfrac{\tan(x)}{x+2^x}$.

solution:

$$\frac{\sec^2(x)(x+2^x)-\tan(x)(1+2^x\ln(2))}{(x+2^x)^2}$$

c) $w(r) = \ln\left(\sqrt{1-r^2}\right)$.

solution:

$$\frac{1}{\sqrt{1-r^2}}\frac{1}{2}(1-r^2)^{-1/2}(-2r)$$

[2] **7.** Use logarithmic differentiation to find the derivative of $f(x) = (x^2+2)^{\cos(x)-1}$. Give your answer in terms of x (not f), but you do not need to simplify.

solution: First take ln of both sides.

$$\begin{aligned}\ln(f) &= \ln\left(\left(x^2+2\right)^{\cos(x)-1}\right)\\ &= (\cos(x)-1)\ln(x^2+2)\end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute f at the end.

$$\frac{1}{f}f' = (-\sin(x))\ln(x^2+2)+(\cos(x)-1)\frac{1}{x^2+2}(2x)$$

$$f' = \left(x^2+2\right)^{\cos(x)-1}\left((-\sin(x))\ln(x^2+2)+(\cos(x)-1)\frac{1}{x^2+2}(2x)\right)$$

[3] **8.** A rectangle is changing width and height continuously. When the width is 20cm and the area is 100cm^2, the width is increasing at a rate of 3cm/s and the area is increasing at a rate of $10\text{cm}^2/\text{s}$. At this moment, what is the rate of change of the height?

solution: Consider a rectangle of width w, height h and area A. We know that $A = wh$. Differentiating we get $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}w}{\mathrm{d}t}h + w\dfrac{\mathrm{d}h}{\mathrm{d}t}$. When the width is 20 and the area is 100 the height must be $100/20 = 5$. At this moment we know $\frac{\mathrm{d}w}{\mathrm{d}t} = 3$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 10$.

$$\begin{aligned}
\frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\mathrm{d}w}{\mathrm{d}t}h + w\frac{\mathrm{d}h}{\mathrm{d}t} \\
10 &= (3)(5) + (20)\frac{\mathrm{d}h}{\mathrm{d}t} \\
\frac{\mathrm{d}h}{\mathrm{d}t} &= \frac{10-15}{20} \\
&= -\frac{1}{4}
\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.