## Review Questions for Midterm 2

MAT1341, Summer 2017

This midterm test covers Chapters 4-10, 14 and 18

Also see Linear Independence Examples and True-False questions posted before.

1. Determine whether each of the following statements is true or false.

(1) If dim $(V) = 3$, then a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in $V$ is linearly dependent.

(2) If a set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in a vector space $V$ with dim $V = 5$, then $S$ is linearly independent.

(3) If a set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in a vector space $V$ with dim $V = 4$, then $S$ is linearly independent.

(4) If a set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in a vector space $V$ with dim $V = 4$, then span $(S) = V$.

(5) If a set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in a vector space $V$ is linearly independent and span $(S) = V$, then dim $(V) = 4$.

(6) If a set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ in a vector space $V$ with dim $(V) = 4$ is linearly independent, then span $(S) = V$.

(7) If vector space $V$ has a basis $B$ with four vectors, then dim $(V) = 4$.

(8) If $S$ is a linearly independent set of a vector space $V$, then there exists a basis $B$ that contains $S$.

(9) If $S$ is a set vectors in a vector space $V$, and span $(S) = V$, then there exists a basis $B$ that contains $S$.

(10) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of a vector space $V$, then $T = \{\mathbf{v}_1 - \mathbf{v}_2 - \mathbf{v}_3, -\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3, -\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3\}$ is also a basis of $V$.

(11) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of a vector space $V$, then $T = \{\mathbf{v}_1, \mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3\}$ is also a basis of $V$.

(12) If $W$ is a subspace of a vector space $V$, then dim $(W) <$ dim $(V)$.

2. True-false questions about matrix multiplication and inversion:

Let $A$, $B$, and $C$ be $n \times n$ matrices, and $c$ is a nonzero scalar. Determine whether each of the following statements is always true:

(1) $AB = BA$.

(2) If the product $AB$ is the zero matrix, then either $A$ or $B$ is a zero matrix.

(3) If $AB = AC$, then $B = C$.

(4) $(A + B)(A - B) = A^2 - B^2$.

(5) $c(A + B)^T = cA^T + cB^T$.

(6) $c(AB) = (cA)(cB)$.

(7) $(cA^{-1}) = cA^{-1}$.

(8) $(cA)^{-1} = cA^{-1}$.

(9) $((AB)^T)^{-1} = (A^T)^{-1}(B^T)^{-1}$.

3. Let $\mathbf{F}$ be the set of all one-variable real-value functions defined on $\mathbb{R}$. Show that the subset $S = \{p \in \mathbf{F} \mid p(1) = p(-1)\}$ is a subspace of $\mathbf{F}$.

4. Show that the set $S = \{(x, y, z) \mid x + y = 2z\}$ is a subspace of $\mathbb{R}^3$.

5. Show that $S = \{(x, y) \mid x - y^2 = 0\}$ is not a subspace of $\mathbb{R}^2$.

6. Show that $S = \{(x, y) \mid x - y = 1\}$ is not a subspace of $\mathbb{R}^2$.

7. For which value(s) of $k$, is the set $S = \{(1, -2, 3), (1, 0, -1), (-1, 3, k)\}$ linearly independent?

8. Let $\mathbf{P}_3$ be the vector space that consists of all polynomials of degree no more than 3. Show that $S = \{1, 1 + x, x + x^2, x^2 + x^3\}$ is a basis of $\mathbf{P}_3$.

9. For which value(s) of $a$, is the vector $(1, 4, 2)$ in span $\{(a, 3, 1), (1, -4, -1)\}$?

10. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -3 \end{bmatrix}$.