Assessment for Learning - Task 1: Knowledge/Understanding questions

A. Expand and simplify the following powers. Give all of the answers in terms of positive exponents. Make sure that you type your solutions using Equation Editor in MS Word or in Open Office Writer (or equivalent).

a. $(3a^4b^3)^2(-5a^7b^2)(4a^3b^6)^0$

Answer:

$(3a^4b^3)^2(-5a^7b^2)\,(4a^3b^6)^0$

$= (9a^8b^6) \times (-5a^7b^2) \times 1$

$= -45a^{15}b^8$

b. $-\left(\frac{x^{4a}}{y^{2a}}\right)^{3b^2}$

Answer:

$-\left(\frac{x^{4a}}{y^{2a}}\right)^{3b^2}$

$= -\frac{x^{12ab^2}}{y^{6ab^2}}$

Brackets are important - don't just get rid of them :)

c. $(-45x^0)+64^{-13}+(2^2)^3$

Answer:

$-(45x^0) + 64^{-\frac{1}{3}} + (2^2)^3$

$= -45 + \frac{1}{4} + 64$

$= \frac{1}{4} + 19$

$= \frac{77}{4}$ or 19.25

Fractions are more accurate to work with than decimals. I recommend you use fractions whenever possible.

d. $b^{(5x-2)} \times b^{(2x+3)}$

Answer:

$b^{(5x-2)} \times b^{(2x+3)}$

$= b^{(5x-2)+(2x+3)}$

$= b^{7x+1}$

e. $\frac{(x^{7a})^{a-b}}{(x^{3a})^{a-2b}}$

Answer:

$$= \frac{x^{7a^2-7ab}}{x^{3a^2-6ab}}$$

$$= x^{(7a^2-7ab)-(3a^2-6ab)}$$

$$= x^{4a^2-ab}$$

B. Evaluate $\log_5 625$ using the following methods.

a. By switching from logarithmic to exponential form

$\log_a c = b \leftrightarrow a^b = c$ a = 5, b = 4, c = 625

Answer:

$5^4 = 625$

b. By cancelling the base with the log base

Answer:

$log_5 5^4 = 4$

c. By using the "change of base" rule

Answer:

$$\frac{log 625}{log 5} = 4$$

C. Simplify and evaluate each of the following expressions using the laws of logarithms.

a. **$\log_6 2 + \log_6 18$**

Answer:

$\log_6 2 + \log_6 18$

$= \log_6 (2 \times 18)$

$= \log_6 36$

$= 2$

b. **$\log_4 256 + \log_4 2 - \log_4 8$**

Answer:

$\log_4 256 + \log_4 2 - \log_4 8$

$= \log_4 512 - \log_4 8$

$= \log_4 (512 \div 8)$

$= \log_4 64$

$= 3$

c. **$\log_2 128 - 3\log_2 4$**

Answer:

$\log_2 128 - 3\log_2 4$

$= \log_2 128 - 3\log_2 4$

$= \log_2 128 - \log_2 4^3$

$= \log_2 128 - \log_2 64$

$= \log_2 (128 \div 64)$

$= \log_2 2$

$= 1$

D. Solve each of the following equations.

a. **$\log_x 81 = 2$**

Answer:

$log_x 81 = 2$

$x^2 = 81$

$x = \pm\sqrt{81}$

$x = \pm 9$

X cannot be -9... why not?

b. **$\log_{12} x = 3$**

Answer:

$log_{12} x = 3$

$12^3 = x$

$x = 1728$

c. **$\log_7 (2x - 5) = 3$**

Answer:

$log_7 (2x - 5) = 3$

$2x - 5 = 7^3$

$2x - 5 = 343$

$2x = 348$

$x = 174$

Task 2: Thinking questions

5. Use systematic trial and error to evaluate log35.

Answer:

$10^1 = 10$

$10^2 = 100$

$10^{1.5} = 31.622$

$10^{1.55} = 35.481$

$10^{1.54} = 34.673$

$10^{1.544} = 34.994$

$10^{1.5441} = 35.002$

6. A culture of bacteria doubles every three hours. The initial culture contains 500 bacteria.

    a. How many bacteria will there be after 27 hours?

Answer:

- *A* is the final number of bacteria
- *I* is the initial number of bacteria
- 2 represents that the population is doubling
- *t* is the time that has elapsed
- *d* is the doubling time

$A = I(2)^{\frac{t}{d}}$

$A = 500(2)^{\frac{27}{3}}$

$A = 500(2)^9$

$A = 256000$

Don't forget to include a concluding statement for every solution. It is your last opportunity to illustrate that you understand how the result satisfies the question.

    b. How long will it take for the culture to reach 16 000 bacteria?

Answer: It will take approximately 15 hours for culture to reach 16000 bacteria.

$16000 = 500(2)^{\frac{x}{3}}$

$32 = 2^{\frac{x}{3}}$

$2^5 = 2^{\frac{x}{3}}$

$5 = \frac{x}{3}$

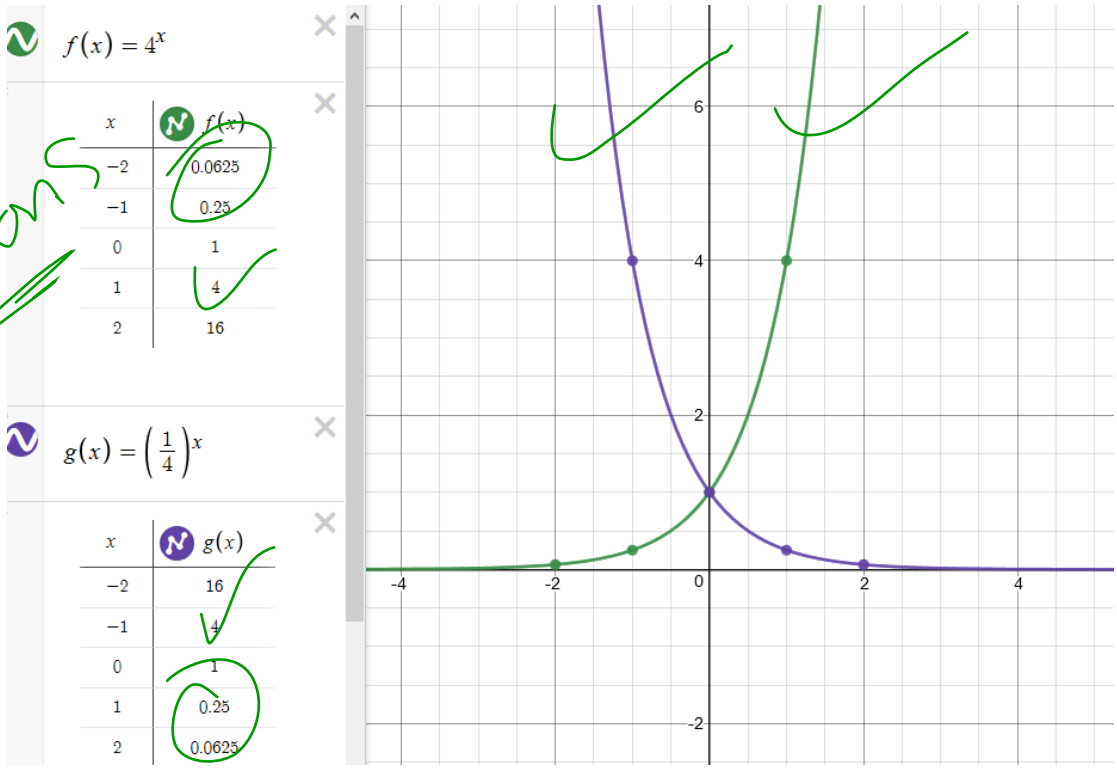
$x = 15$

7. Complete the following comparison of two functions.

    a. Create a table of values for each exponential function: $y=4^x$ and $y=(1/4)^x$. Use x-values of 2, 1, 0, –1, and –2. Then, graph the

equations on the same grid. Include each table of values and graph in your solution.

Answer: f(x) = 4^x and g(x) = $(1/4)^x$



b. Indicate which graph represents exponential growth and which represents exponential decay.

Answer:

The graph of $y = 4^x$ represents an **exponential growth curve** because the graph *increases* at an accelerated rate as it moves from left to right.

The graph of $y = (1/4)^x$ represents an **exponential decay curve** because the graph *decreases* at a decelerated rate as it moves from left to right.

c. List the similarities and differences between the graphs.

Answer:

Similarities

- There are no *x*-intercepts.
- There is a horizontal asymptote at $y=0$.
- The *y*-intercepts are both (0,1).
- There are no maximum or minimum points.
- The domain is all real numbers.
- The range is all real numbers greater than 0.

Differences

- The graph of $y=4^x$ increases, while the graph of $y=(1/4)^x$ decreases.
- The graphs are reflections of each other in the y-axis.
- The bases in the equations $y=4^x$ and $y=(1/4)^x$ are reciprocals.

Task 3: Communication questions

8. Explain the difference between the exponential form, the radical form, and the logarithmic form of an equation, and give an example of each.

Answer:

Exponential form is made up of a base and an exponent. For example, in the power 3^2, "3" is the base and "2" is the exponent. The exponent tells us how many times the base is being multiplied by itself. In our case is twice or 3 x 3.

Example: $3^2 = 3 \times 3 = 9$

Radicals or roots refer to the opposite operation of applying exponents. For instance, if we square 2, we get 4, and if we take the "square root of 4", we get 2. So, in general we are trying to isolate the base.

Example: So the exponential form $2^2 = 4$, becomes $4^{\frac{1}{2}} = 2$ or $\sqrt{4} = 2$

Logarithmic form involves the exponent of the power. We can find an exponent by changing the equation to logarithmic form. The equation $3^3 = 27$ can be rewritten as a logarithm: $x = \log_3 27$.

Example: $log_3 27 = 3$

9. Define a common logarithm.

Logarithms with base 10 are called common logarithms. For instance log1000 is the same as $\log_{10}1000$.

Example: $log 1000 = log_{10} 1000 = 3$

What is unique about these?

10. A fellow classmate is having difficulty solving the following expression involving logarithms. List the steps you would take to solve for "x."

    $\log_3 9+\log_3 2x^2=\log_3 162$

Answer:

$log_3 9 + log_3 2x^2 = log_3 162$

$log_3 18x^2 = log_3 162$

$18x^2 = 162$

$x^2 = 9$

$x = 3$

This type of question expects you to do the calculations and to write out the reason for and explanation of each step for your classmate. When doing the communication tasks, more is usually better and you will want to use as many communication tools as are appropriate for the task so that you don't miss anything (graphs, equations, diagrams, calculations, examples, sentences, ...)

11. Explain why there is no solution for the equation $\log_2 0 = x$. Use a graph to justify your answer.

Answer:

The equation is undefined as there is no values that can make $2^x = 0$

Why? Prove it.

Where is the graph expected for this question?

12. Use Equation Editor to correctly type and evaluate the expression: common log of the square root of $\frac{9}{25}$. Round your answer to two decimal places.

Answer:

$$log\frac{9}{25}$$
$$= log9 - log25$$
$$= 0.95 - 1.39$$
$$= 0.95 - 1.39$$
$$= 0.44$$

Task 4: Application questions

13. The pH of coffee is 5.2 and the pH of orange juice is 4.4. How many times more acidic is orange juice than coffee? Round your answer to one decimal place.

Answer: Orange juice is 6.3 times more acidic than coffee

$For\ pH\ of\ 4.4, we\ get [H^+] = 10^{-4.4}$

$For\ pH\ of\ 5.2, we\ get [H^+] = 10^{-5.2}$

$$\frac{10^{-4.4}}{10^{-5.2}} = 6.3$$

Concluding statement and show all of your thinking and calculations.

These would be 10 to 12 mark questions, you need to prove that many unique thoughts to earn those marks.

14. An earthquake with a magnitude of 5.4 is 39 times as intense as an aftershock that occurs five hours later. What is the magnitude of the aftershock? Round your answer to one decimal place.

Answer: The magnitude of the aftershock is 3.8

$$M = \log\left(\frac{10^{5.4}}{39}\right) = 3.8$$

15. The intensity of the sound of a jet engine is 140 dB, while the intensity of the sound of a motorcycle is 100 dB. How many times louder is a jet engine than a motorcycle?

Answer: The sound of a jet engine is 10000 times louder than a motorcycle

$$10\log\left(\frac{I_1}{I_0}\right) = 140$$

$$\log\left(\frac{I_1}{I_0}\right) = 14$$

$$\frac{I_1}{I_0} = 10^{14}$$

$$I_1 = I_0 \times 10^{14}$$

And

$$10\log\left(\frac{I_2}{I_0}\right) = 100$$

$$\log\left(\frac{I_2}{I_0}\right) = 10$$

$$\frac{I_2}{I_0} = 10^{10}$$

$$I_2 = I_0 \times 10^{10}$$

Dividing the most intense sound to the least intense, we get:

$$\frac{I_2}{I_1} = \frac{I_0 \times 10^{14}}{I_0 \times 10^{10}} = 10^{14-10} = 10^4 = 10000$$

Concluding statement.

16. A new car worth $18 000 depreciates at a rate of 8% per year. Round your answers for the following questions to the nearest whole number.

    a. What will the car be worth after four years?

Answer: The car will be worth 12895$ after 4 years

$$A = 18000(1 - 0.08)^4 = 12895$$

Don't forget the proper units $$$ and the concluding statement and all the calculations

    b. When will the car be worth half of its value?

Answer: The car will be worth half its value in 8.3 years

$$9000 = 18000(1 - 0.08)^n$$

$$0.5 = (0.92)^n$$

$$log0.5 = log0.92^n$$

$$log0.5 = nlog0.92$$

$$n = \frac{log0.5}{log0.92} \cong 8.3$$