University of Waterloo

Department of Economics

ECON 391 (Equilibrium in Market Economies)

Assignment #1 Due: Tuesday, 29 January, 2019

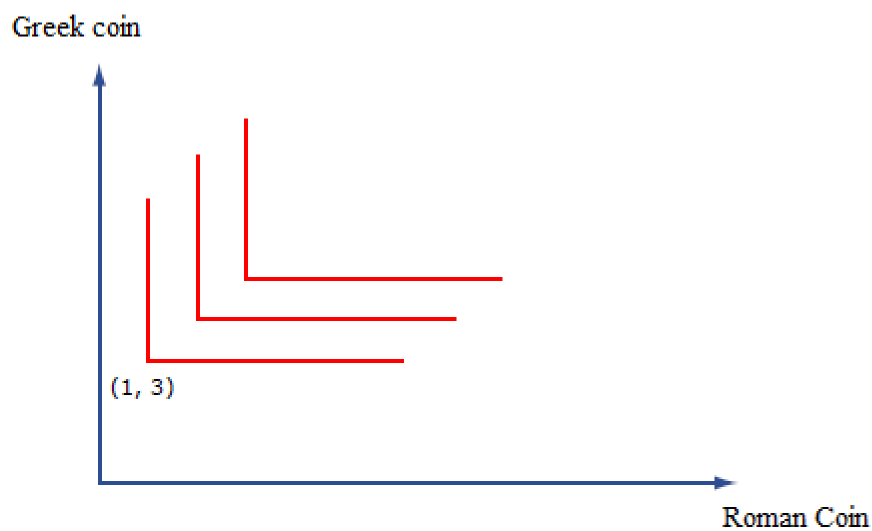
You can work individually or in groups of to 4 people maximum. If you prefer to work in a group, please submit just one copy for the group with the Names and Student IDs of each person.

Total points you can get is 20.

1. ***(6 points)*** Tom decides to be a collector of Ancient Greek (g) and Ancient Roman (r) coins. He wants to have matched sets of three Greek coins and one roman coin , and he doesn't want to have any Greek coins or Roman coins that are not in a matched set of this size.

   (a) ***(1 point)*** Draw Tom's indierence curves and write his utility function. Put Greek coins on the y axis and Roman coins on the x axis. Assume he receives utility of 3 utils from each matched coin set he has.

   $$U(g, r) = 3 \min \{3r, g\} \text{or } U(g, r) = 3 \min \left\{r, \tfrac{g}{3}\right\}$$

   (b) ***(1 point)*** Currently, Tom has 32 dollars to spend. The price of Roman coin is $p_r = 2$ and the price of Greek coin is $p_g = 2$. What is the optimal allocation of Greek and Roman coins for Tom? How much utility does he achieve from this allocation?

   We can find the optimal allocation by nding the intersection of the lines$g = 3r$ and $2g + 2r = 32$. Substituting in to the budget constraint, this yields the following.

   $$6r + 2r = 32$$

   $$r* = 4.$$

   We have $g^* = 12$ and $U(r, g) = 3 \min \{3r^*, g^*\} = 3 \min \{12, 12\} = 3 \cdot 12 = 36$

(c) ***(1 point)*** Due to a shortage at the antiques market, the price of Roman coins increases to $p_r = 10$. What is the new allocation of Greek and Roman coins at this price level, and what utility does Tom obtain?

Re-solve the problem with a new budget constraint $2g + 10r = 32$. This yields

$$6r + 10r = 32$$

$$r^* = 2,$$

We have $g^* = 6$ and $U(r, g) = 3 \min\{3r^*, g^*\} = 3 \min\{6, 6\} = 3 \cdot 6 = 18$

(d) ***(1 point)*** Luckily, Tom's parents value their son's utility, and are willing to give him enough income to ensure that he has the same utility he did prior to the price change. How much extra money do they have to give him?

Tom's parents have to give him sufficient income to buy two more matched sets at the new prices. This requires the purchase of two Roman coins and six Greek coins, at a cost of $2 \cdot 10 + 6 \cdot 2 = 32$. They have to double his income, increasing it by \$32, in order to maintain him at the same utility level.

(e) ***(1 point)*** Tom has friend Jack who doesn't like matched sets of coins and has different preferences. His utility function is rg^2. If he started with the same coin budget as Tom of 32 dollars and then faced the same price shock, what would be the decrease in his utility when the price of Roman coin increases from \$2 to \$10?

First, solve for Jack's original optimal allocation. We set the ratio of marginal utilities equal to the ratio of prices.

$$\frac{g^2}{2rg} = \frac{2}{2}$$

$$g = 2r$$

Substituting into the budget constraint, this yields

$$2(2r) + 2r = 32$$

This yields $r = \frac{16}{3}$, $g = \frac{32}{3}$, and $u(r, g) = \frac{16}{3}\left(\frac{32}{3}\right)^2 = 606.814$

When prices increase, we now re-solve the problem.

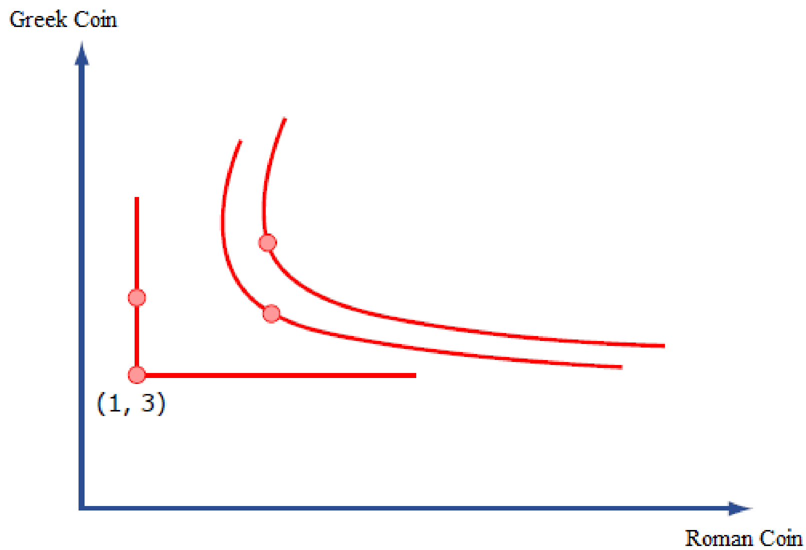
$$\frac{g^2}{2rg} = \frac{10}{2}$$

$g = 10r$ Substituting into the budget constraint, this yields $2(10r) + 10r = 32$. This yields $r = \frac{16}{15}$, $g = \frac{160}{15}$, and $u(r, g) = \frac{16}{15}\left(\frac{160}{15}\right)^2 = 121.362$.

To calculate the decrease in utility, subtract the second level of utility from the first. That yields 485.452. Jack experiences a 80% decline in his utility.

(f) ***(1 point)*** Tom and Jack's parents are going to give a gift of equal monetary value to both friends. They are trying to decide whether to give cash or give coin. Which friend is more likely to prefer cash?

Tom is more likely to prefer cash, because he only receives utility from a gift that is given as a matched set and no utility from any other type of gift. You can portray this graphically by showing a graph of the two utility functions and indicating that for Tom, any increase in the quantity of one good or the other keeps her on the same indierence curve, while for Jack, any increase will move her to a new indifference curve. The below graph shows the result of a gift of Roman coin only to both of them. Tom is on the same utility curve, but Jack has increased utility. In this case and others like it, Tom is more likely to prefer cash.



2. ***(5 points)*** Consider the following problem:

$$\max_{x,y} u(x,y) = x^{\alpha} y$$

$$s.t.\, p_x x + p_y y = 10,\; x \geq 0, y \geq 0$$

(a) ***(3 points)*** Find demand functions $x^*(p_x, p_y \alpha)$ and $y^*(p_x, p_y \alpha)$
Because we have a Cobb Douglas function, we will have an interior solution where $MRS = \frac{p_x}{p_y} \rightarrow \frac{\alpha y}{x} = \frac{p_x}{p_y}$. , we have This yields $x^*(p_x, p_y \alpha) = \frac{\alpha}{\alpha+1}\frac{10}{p_x}$, and $y^*(p_x, p_y \alpha) = \frac{1}{\alpha+1}\frac{10}{p_y}$.

(b) ***(2 points)*** Define the condition for α to satisfy $x \geq 0$ and $y \geq 0$ if $p_x > 0$ and $p_y > 0$.
For $p_x > 0$ and $p_y > 0$, we have $\frac{\alpha}{\alpha+1}\frac{10}{p_x} \geq 0\, and\, \frac{1}{\alpha+1}\frac{10}{p_y} \Leftrightarrow \alpha \geq 0$.

3. ***(4 points)*** For each of the following utility functions, find the optimal consumptions bundles using the standard budget constraint $p_1 x_1 + p_2 x_2 = m$.

(a) ***(1 point)*** $u(x_1, x_2) = \min(2x_1, 3x_2)$
$2x_1 = 3x_2 \rightarrow x_2 = \frac{2}{3}x_1$ plugging into the budget line, we get $p_1 x_1 + p_2 \frac{2x_1}{3} = m \rightarrow x_1\left(\frac{3p_1+2p_2}{3}\right) = m$. This yields $x_1^* = \frac{3m}{3p_1+2p_2}$ and $x_2^* = \frac{2m}{3p_1+2p_2}$.

(b) ***(1 point)*** $u(x_1, x_2) = 2x_1 + 3x_2$
$MRS = \frac{2}{3}$. If $\frac{p_1}{p_2} > \frac{2}{3}$ then $x_1^* = 0$ and $x_2^* = \frac{m}{p_2}$. If $\frac{p_1}{p_2} < \frac{2}{3}$ then $x_1^* = \frac{m}{p_1}$ and $x_2^* = 0$. If $\frac{p_1}{p_2} = \frac{2}{3}$ then any bundle will be optimal choice.

(c) ***(1 point)*** $u(x_1, x_2) = x_1^{1/2} x_2^{1/3}$
$MRS = \frac{3}{2}\frac{x_2}{x_1} = \frac{p_1}{p_2} \rightarrow x_2 p_2 = \frac{2}{3}x_1 p_1$. Plugging into the budget line, $p_1 x_1 + \frac{2}{3}p_1 x_1 = m$. That yields $x_1^* = \frac{3m}{5p_1}$ and $x_2^* = \frac{2m}{5p_2}$.

(d) ***(1 point)*** $u(x_1, x_2) = \frac{1}{2}\ln x_1 + \frac{1}{3}\ln x_2$
$MRS = \frac{3}{2}\frac{x_2}{x_1} = \frac{p_1}{p_2} \rightarrow x_2 p_2 = \frac{2}{3}x_1 p_1$. Plugging into the budget line, $p_1 x_1 + \frac{2}{3}p_1 x_1 = m$. That yields $x_1^* = \frac{3m}{5p_1}$ and $x_2^* = \frac{2m}{5p_2}$.

4. ***(10 points)*** Mary has 24 hours before her microeconomics midterm. She has a econometrics midterm directly after the micro midterm and has no time to study in between. Her utility function is given by

$$u(e,m) = e^{0.6}m^{0.4}$$

where e is the score on the econometrics midterm and m is the score on the microeconomics midterm. Although she cares more about econometrics, she is better at microeconomics; for each hour spent studying microeconomics she will increase her score by 3 points, but her econometrics score will only increase by 2 points for every hour spent studying econometrics. Studying zero hours results in a score of zero on both subjects.

(a) ***(2 points)*** What constraints does Mary face in her test score maximization problem?

Assume that h_e is the hour that she spends for econometrics and h_m is the hour to spend for microeconomics. The constraint that she faces is $h_e + h_m \leq 24$ and $h_e \geq 0$ and $h_m \geq 0$. We can consider $e = 2h_e$and $m = 3h_m$

(b) ***(2 points)*** How many hours should Mary optimally spend studying econometrics? How many hours studying microeconomics? (hours are divisible)

$\max_{h_e} (2h_e)^{0.6} (3h_m)^{0.4}$ where $h_m = 24 - h_e$. This yield $\max_{h_e} (2h_e)^{0.6} (3(24-h_e))^{0.4}$. Making the monotonic transformation of the utility fuinction we get $\max_{h_e} 0.6\ln(2h_e) + 0.4\ln(3(24-h_e))$. From the first order condition, we get $0.6\frac{2}{2h_e} - 0.4\frac{3}{3(24-h_e)} = 0$. That yields $\frac{3}{h_e} = \frac{2}{24-h_e}$, $h_e^* = 14.4$ and $h_m^* = 24 - h_e^* = 9.6$

(c) ***(2 points)*** What microeconomics and econometrics test scores will she achieve (i.e. what are m^* and e^*)?

$m^* = 3h_m^* = 3 \cdot 9.6 = 28.8$ and $e^* = 2h_e^* = 2 \cdot 14.4 = 28.8$

(d) ***(2 points)*** What utility level will she achieve?

$u(e,m) = e^{0.6}m^{0.4} = 28.8^{0.6}28.8^{0.4} = 28.8$

(e) ***(2 points)*** Suppose Mary can get an microeconomics tutor. If she goes to the tutor, she will increase her microeconomics test score by 5 points for every hour spent studying instead of 3 points, but will lose 4 hours of study time by going to the tutor. She cannot study while at the tutor, and going to the tutor does not directly improve her test score. Should Mary go to the tutor?

$e = 2h_e$ and $m = 5h_m$ but the constraint will be $h_e + h_m = 20$. Optimization will be $\max_{h_m} 0.6\ln(2(20-h_m)) + 0.4\ln(5h_m)$. From thwe F.O.C. $0.6\frac{1}{20-2h_m} = 0.4\frac{1}{h_m}$. That yields $h_m = 8$, $h_e = 20 - h_m = 12$. $m = 40$, $e = 24$, $u(24, 40) = 29.44$.

Mary has higher utility when she goes to the tutor, so she should go.