MAT 2377C
Midterm-A

17 October 2018 **Professor: Mohsen Rezapour**
Time: 70 minutes

Student Number: _____

Family Name: _____

First Name: _____

This is a closed book examination.
Only non-programmable and non-graphic calculators are permitted.
Record your answer to each question in the table below.
Your package includes the title page, two pages with questions, a formula sheet and statistical tables.
Number of questions: **10**.
NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.
Shade **ONE** letter for each question. A question with more than one shading answer will not be marked.

Question	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

GOOD LUCK !!!

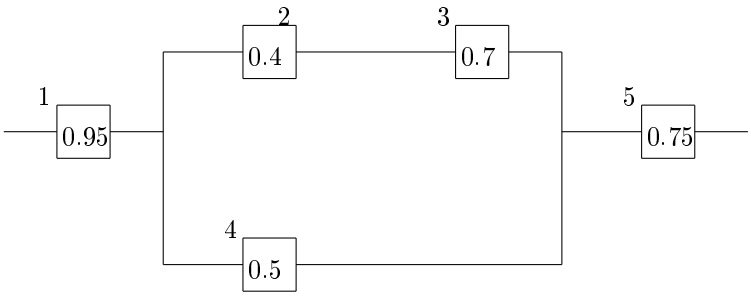
Q1. A president and two treasurers are to be chosen from a student club consisting of 50 people. What is the probability that Jack will select as a president?

(a) 2/50 (b) 49/50 (c) 1/50 (d) 48/50 (e) none of the preceding

Solution to Q1:

The total number is $\frac{50!}{1!2!47!} = 50 * 49 * 48/2$. The number of way that Jack selected as a president is $\frac{49!}{2!47!} = 49 * 48/2$. The probability is $\frac{49 * 48/2}{50 * 49 * 48/2} = 1/50$.

Q2. Consider the following system with five components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown below. Assume that the components function or fail independently. What is the probability that the system works?



(a) 0.814 (b) 0.456 (c) 0.305 (d) 0.506 (e) none of the preceding

Solution to Q2:

Call 'Box B' - components 2,3,4, 'Box C' - components 2,3.

$$\begin{aligned} P(\text{Box C operates}) &= P(\text{component 2 operates and component 3 operates}) \\ &= P(\text{component 2 operates})P(\text{component 3 operates}) = 0.4 \times 0.7 = 0.28. \end{aligned}$$

$$\begin{aligned} P(\text{Box B operates}) &= P(\text{Box C operates or component 4 operates}) \\ &= P(\text{Box C operates}) + P(\text{component 4 operates}) - \\ &\quad P(\text{Box C operates})P(\text{component 4 operates}) \\ &= 0.28 + 0.5 - 0.28 * 0.5 = 0.64. \end{aligned}$$

$$\begin{aligned} P(\text{system operates}) &= P(\text{component 1 and Box B and component 5 operate}) \\ &= P(\text{component 1 operates})P(\text{Box B operates})P(\text{component 5 operates}) \\ &= 0.95 * 0.64 * 0.75 = 0.456. \end{aligned}$$

Answer **B**.

Q3. Let X denote a number of automobile that have slight paint blemishes receiving by an agency during a month. Its probability mass function is given by

x	1	2	3	4	5	6
$P(X=x)$	0.12	0.15	0.14	0.13	0.18	0.28

- The probability that there are less than 3 paint blemishes automobile receive by the agency within a month, and
- the mean of X

are, respectively

(a) 0.28; 2.50 (b) 3.26; 0.40 (c) 0.59; 2.32

(d) 0.27; 3.94 (e) none of the preceding

Solution to Q3:

(a) To compute

$$P(X<3) = P(X=1) + P(X=2) = 0.27\,.$$

(b)

$$E(X) = 1\times 0.12 + 2\times 0.15 + \cdots + 6\times 0.28 = 3.94\,.$$

Answer **D**.

Q4. A manufacture of a flue vaccine is concerned about the quality of its flue serum. The produced serums are divided into two equal-size parts. The items of the first and second batches are processed by department A and B, respectively. The probability that a serum is rejected by department A is 0.1 and the probability for department B is 0.08. What is the probability that a selected serum after the processing is accepted?

(a) 0.85 (b) 0.51 (c) 0.91

(d) 0.93 (e) none of the preceding

Solution to Q4:

Denote the events: C - the company A process the serum; D - the company B process the serum, R- the serum is rejected. We have $P(C) = P(D) = 0.5$; $P(R \mid C) = 0.1$, $P(R \mid D) = 0.08$. We calculate first, using the total probability rule,

$$P(R) = P(R \mid C)P(C) + P(R \mid D)P(D) = 0.5 * 0.1 + 0.5 * 0.08 = 0.09\,.$$

Hence, $P(C^c) = 1 - 0.09 = 0.91$. Answer **C**.

Q5. The probability that a costumer entering a certain shopping center buys the item A is 0.15. The probability that the third costumer is the the first person buying the item A equals (Hint: geometric)

(a) 0.6068 (b) 0.3932 (c) 0.9345

(d) 0.1083 (e) none of the preceding

Solution to Q5:

success = costumer by more than \$ 1000, $p = P(\text{succes}) = 0.15$. Let X be the number of costumer until we have one that buy item A. Then X has geometric distribution with the parameter p. We are looking for

$$P(X=3) = (1-p)^2 p = 0.1083\,.$$

Answer **D**.

Q6. Consider a random variable X with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x \leq -1 \\ \frac{1}{9}(x^3+1) & \text{if } -1 < x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

What is the $P(X \leq 1.5)$?

(a) 0 (b) 0.4861 (c) 0.5487

(d) 1 (e) none of the preceding

Solution to Q6:

To compute:

$$P(X \leq 0.5) = \frac{1}{9}((1.5)^3+1) = 0.4861$$

Answer **B**.

Q7. A lot contains 50 missiles, including 8 defective that will not fire. We choose randomly 5 missiles. What is the probability that all 5 selected will not fire?

(a) 0.4015 (b) 0.5895 (c) 0.3725

(d) 0.4275 (e) none of the preceding

Solution to Q7:

This is not the binomial experiment: Let X be the number of defective fuses. We calculate

$$P(X=5) = \frac{\binom{8}{5}\binom{42}{0}}{\binom{50}{5}} = 56/2118760 \approx 0\,.$$

Answer **E**.

Q8. Past experience suggests that there are, on average, 2 potholes per mile on a highway after a certain amount of usage satisfying the Poisson process conditions. What is the probability that no potholes will occur in a given section of 5 miles?

(a) $e^{-1/10}$ (b) $e^{-2/5}$ (c) e^{-2} (d) e^{-10} (e) none of the preceding

Solution to Q8:

Let X be a Poisson random variable with parameter $\lambda = 2$ and $t = 5$. To compute: $P(X=0) = \exp(-\lambda t) = \exp(-10)$. Answer **D**.

Q9. A certain type of storage battery lasts, on average, 3 years with standard deviation of 0.5 years. Assuming that battery life is normally distributed, find the probability that the given battery will last between 2 to 4 years.

(a) 1 (b) 0.9544 (c) 0.4560 (d) 0.9772 (e) none of the preceding

Solution to Q9:

$$P(2 < X < 4) = P((2-3)/0.5 < Z < (4-3)/0.5) = P(-2 < Z < 2) = 0.9772 - 0.0228 = 0.9544$$

Answer **B**.

Q10. It is known, from previous data, that the number of car entered a carwash per 1-hour has Poisson distribution with parameter $\lambda = 2$. What is the probability that the second car will enter the carwash after three hours?

(a) $3e^{-6}$ (b) $e^{-4}4^3/3!$ (c) $7e^{-6}$ (d) $e^{-6}6^2/2!$ (e) none of the preceding

Solution to Q10:

The time to see the second car has gamma distribution with parameter $\alpha = 2$ and $\beta = 1/\lambda = 1/2$ and density function $f(x;\alpha,\beta) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$. Thus,

$$P(X > 3) = \int_3^\infty f(x;\alpha,\beta)dx = \int_3^\infty \frac{2^2}{1!} x e^{-2x} dx = 4\int_3^\infty x e^{-2x} dx = 7e^{-6}$$

Answer **C**.

This is the last question

Solutions to multiple choice questions:

Q1 $\longrightarrow$ c

Q2 $\longrightarrow$ b

Q3 $\longrightarrow$ d

Q4 $\longrightarrow$ c

Q5 $\longrightarrow$ d

Q6 $\longrightarrow$ b

Q7 $\longrightarrow$ e

Q8 $\longrightarrow$ d

Q9 $\longrightarrow$ b

Q10 $\longrightarrow$ c