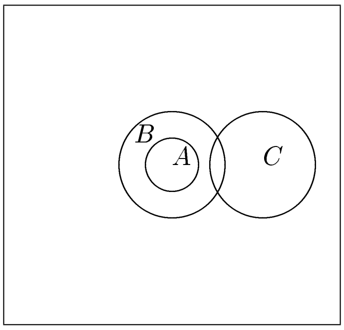
Assignment 1
Due date: 1 October 2018
Note: You have to drop off your assignment in the class. It is advised that you make a copy of your assignment.
Note: I will post solutions to all questions. However, only selected questions will be marked.

Q1. The three events are shown on the Venn diagram:



Reproduce the figure and shade the region corresponding to the following events:

(a) $(A' \cap B) \cup (A' \cap B')$ (b) $(A \cap B) \cup C$ (c) $A \cap (B \cup C)'$

(d) $(A \cap B)' \cup C$ (e) $(A \cap C)' \cup B$

Solution to Q1:

The shaded region should be
(a) A' (b) $A \cup C$ (c) $\emptyset$ (d) A' (e) S

Marking scheme for Q1:

1 point for each correct answer. Total - 5 points.

Q2. In a group of 16 candidates for laboratory research positions, 6, 5, and 5 of candidates are chemists, physicians, and biochemists, respectively. In how many ways one can choose 2 chemists, 3 physicians, and 3 biochemists,
(a) if we have no further condition.
(b) if Rose and Jack (who are biochemists) are elected together or are not elected at all?

Solution to Q2:

a)

$$\binom{6}{2} * \binom{5}{3} * \binom{5}{3}.$$

b)

$$\binom{6}{2} * \binom{5}{3} * \binom{3}{1} + \binom{6}{2} * \binom{5}{3} * \binom{3}{3}.$$

Marking scheme for Q2:

(This question will not be marked)

Q3. An automobile manufacturer is concerned about a possible recall of its best-selling four-door sedan. If there was a recall, there is a probability of 0.25 of a defect in the brake system, 0.18 of a defect in the transmission system, 0.07 of a defect in both systems simultaneously, 0.17 of a defect in the fuel system, 0.22 of a defect in the lighting system, and 0.30 of a defect in some other area.

(a) What is the probability that the defect is not in the brake system?

(b) What is the probability that the defect is in the brake system or in the transmission system?

(c) What is the probability that there are no defects in either the brake or transmission system?

(d) What is the probability that the defect is in the brake system but not in the transmission system?

(e) What is the probability that the defect is in the fueling system or in some other area, if we know that these events are exclusive?

(f) What is the probability that the defect is in the brake system, if the automobile has a defective in the transmission system?

(g) What is the probability that the defect is in the brake system, if the automobile does not have defective in the transmission system?

(h) Suppose that a fueling broken automobile has a defective in lighting system with probability 0.22, then what is the probability that a lighting broken automobile has a defective in fuel system?

(i) Are the two events in part (h) independent?

Solution to Q3:

Let A be the event of defect in the brake system, B be the event of defect in the transmission system, C be the event of defect in the fuel system, D be the event of defect in the lighting system, and E be the event of defect in some other area.

(a) $P(A') = 0.75$

(b) $P(A \cup B) = 0.25 + 0.18 - 0.07 = 0.36$

(c) $P((A \cup B)') = 1 - P(A \cup B) = 0.64$

(d) $P(A \cap B') = P(A) - P(A \cap B) = 0.25 - 0.07 = 0.18$

(e) $P(C \cup E) = P(C) + P(E) = 0.17 + 0.30 = 0.47$

(f) $P(A|B) = P(A \cap B)/P(B) = 0.07/0.18 = 0.38$

(g) $P(A|B') = P(A \cap B')/P(B') = 0.18/0.82 = 0.219$

(h) $P(D|C) = P(D)$ therefore C and D are independent and $P(C|D) = P(C) = 0.17$

(i) Yes they are independent

Marking scheme for Q3:

1 point for each correct answer. Total - 9 points.

Q4. In tossing two dies problem consider the events $A = \{$ the first outcome be odd and the second outcome is greater than 5$\}$, $B = \{$ the two outcomes are the same$\}$, and $C = \{$ the sum of two outcomes is less than four $\}$, and obtain the following probabilities.

(a)	$P(A \cup B \cup C)$	(b)	$P(A \cap B \cap C)$	(c)	$P(A \cap B)$
(d)	$P((A \cup B) \cap C)$	(e)	$P(A' \cap B' \cap C')$	(f)	$P[(A \cup B \cup C)']$

Solution to Q4:

$A = \{(1,6),(3,6),(5,6)\}$, $B = \{(1,1),\ldots,(6,6)\}$, and $C = \{(1,1),(1,2),(2,1)\}$. Then the events are mutually exclusive. We have $P(A) = P(C) = 3/36$ and $P(B) = 6/36$.

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) = 3/36 + 6/36 + 3/36 - 0 - 0 - 1/36 + 0 = 11/36$.

(b) $P(A \cap B \cap C) = 0$

(c) $P(A \cap B) = 0$

(d) $P((A \cup B) \cap C) = P(\{(1,1)\}) = 1/36$

(e) $P(A' \cap B' \cap C') = P[(A \cup B \cup C)'] = 1 - P(A \cup B \cup C) = 25/36$ (draw Venn diagram)

(f) $P[(A \cup B \cup C)'] = 1 - P(A \cup B \cup C) = 25/36$

Marking scheme for Q4:

(This question will not be marked)

Q5. Probability that a electric burner, which is kept in dryness, fails during the guarantee period, is 1%. If the burner is humid, the failure probability is 8%. Assume that 90% of electronic stove are kept in dry conditions, whereas remaining 10% are kept in humid conditions.

(a) What is the probability that the burner fails during the guarantee period?

(b) If the burner failed during the guarantee period, what is the probability that it was kept in humid conditions?

Solution to Q5:

Let F-"failure", H-"humid", D-"dry". Given: $P(F|D) = 0.01$, $P(F|H) = 0.08$, $P(D) = 0.9$, $P(H) = 0.1$

(a)

$$P(F) = P(F|D)P(D) + P(F|H)P(H) = 0.01 * 0.9 + 0.08 * 0.1 = 0.009 + 0.008 = 0.017$$

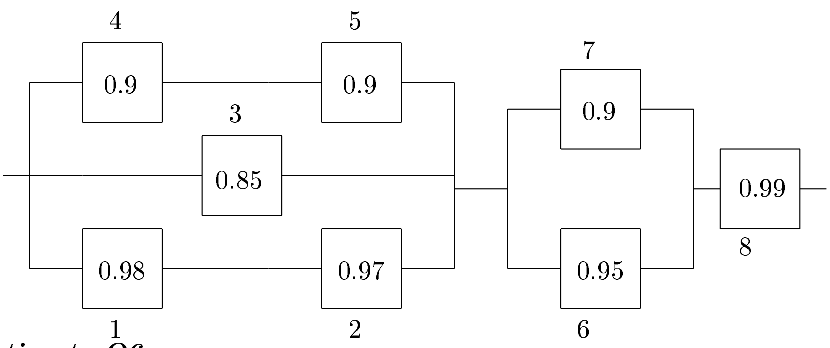
(b)

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F|H)P(H)}{P(F)} = 0.4706$$

Marking scheme for Q5:

1 point for each part. Total - 2 points.

Q6. The following system operates only if there is a path of functional device from left to the right. The probability that each device functions is as shown. What is the probability that the circuit operates? Assume independence.



Solution to Q6:

Let Box A: components 1,2,3,4,5; Box B: components 6,7; Box C: component 8. Let $D_i = \{$ the component ith work $\}$, $i = 1, \ldots, 8$.

$$P(\text{system works}) = P(\text{A works})P(\text{B works})P(\text{C works}).$$

We have

$$P(\text{A works}) = P((D_1 \cap D_2) \cup (D_3) \cup (D_4 \cap D_5))$$

Let $E_1 = D_1 \cap D_2$, $E_2 = D_3$ and $E_3 = D_4 \cap D_5$. Then

$$P((D_1 \cap D_2) \cup (D_3) \cup (D_4 \cap D_5)) = P(E_1 \cup E_2 \cup E_3)$$
$$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

Therefore,

$$P(\text{A works}) = P((D_1 \cap D_2) + P(D_3) + P(D_4 \cap D_5) - P((D_1 \cap D_2) \cap (D_3)) - P((D_3) \cap (D_4 \cap D_5))$$
$$-P((D_1 \cap D_2) \cap (D_4 \cap D_5)) + P((D_1 \cap D_2) \cap (D_3) \cap (D_4 \cap D_5))$$
$$= P(D_1)P(D_2) + P(D_3) + P(D_4)P(D_5) - P(D_1)P(D_2)P(D_3) - P(D_3)P(D_4)P(D_5)$$
$$-P(D_1)P(D_2)P(D_4)P(D_5) + P(D_1)P(D_2)P(D_3)P(D_4)P(D_5) = 0.9985$$

Now, B is just parallel system, so that

$$P(\text{B works}) = 0.9 + 0.95 - 0.9 * 0.95 = 1.85 - 0.855 = 0.995.$$

Final answer: 0.9836.

Marking scheme for Q6:

Total - 3 points.

Q7. The quality control section of a manufacturing company could identify the defective items with probability 0.95 and the probability of incorrectly classifying a good item as defective is 0.02. The company has evidence that its line produces 1% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

(b) If an item selected at random is classified as non-defective, what is the probability that it is indeed good?

Solution to Q7:

Let A - the event that an item is classified as defective, D - the event that an item is defective; so that D^c is the event that an item is 'good'. What is known is: $P(D) = 0.01$; $P(A|D) = 0.95$, $P(A|D^c) = 0.02$.

(a)

$$P(A) = P(A \cap D) + P(A \cap D^c) = P(A|D)P(D) + P(A|D^c)P(D^c) \approx 0.0293.$$

(b) To compute $P(D^c|A^c)$. From Bayes' formula:

$$P(D^c|A^c) = \frac{P(A^c|D^c)P(D^c)}{P(A^c)} = \frac{(1 - P(A|D^c))P(D^c)}{1 - P(A)} \approx 0.999$$

Marking scheme for Q7:

(This question will not be marked)