Assignment III

Due date: 14 November 2018

Note: You have to drop off your assignment in the class. It is advised that you make a copy of your assignment.

Note: I will post solutions to all questions. However, only selected questions will be marked.

Note: Assignments after the due time will not be accepted.

Note: The electronic version of your assignment will not be accepted.

Q1. Two distinct solid fuel propellants, type A and Type B, are being considered for a space program activity. From past data we know that the standard deviation of burning rate of two propellants are the same and equal to 5 cm/sec and the means of the burning rate for propellant A and propellant B equal 20.5 cm/sec and 22.5 cm/sec, respectively. Random sample of 21 specimens of each propellant are taken.

(a) What is the probability that the mean of the burning rate of sample of propellant A is between 19 cm/sec and 22 cm/sec?

(b) What is the probability that the difference between two sample means of burning rates are less than 0.5?

(c) What is the probability that the ratio S_1^2/S_2^2 is less than 2.124, where S_1^2 and S_2^2 are sample variance of the burning rate of specimens of propellant A and B, respectively?

(d) If we assume that the variance of the burning rate of propellant A, i.e. σ_1^2 is unknown and the corresponding sample variance equals 36 cm^2/sec^2, what is the probability that the sample mean of the burring rate of propellant A exceed 23.80993?

Solution to Q1:

Let X be the burning rate of propellant A and Y be the burning rate of propellant B.

(a) We have $\mu_1 = 20.5$ and $\sigma_1 = 5$, $n = 21$ then

$$P(19 < \bar{X} < 22) = P\left(\sqrt{n}\frac{19-\mu_1}{\sigma_1} < \sqrt{n}\frac{\bar{X}-\mu_1}{\sigma_1} < \sqrt{n}\frac{22-\mu_1}{\sigma_1}\right)$$
$$= P(-1.37 < Z < 1.37) = 0.9147 - 0.0853 = 0.9147 - 0.0853$$

(b) We have $\mu_1 = 20.5$, $\mu_2 = 22.5$ and $\sigma_1 = \sigma_2 = 5$, $n = m = 21$ then

$$P(-0.5 < \bar{X} - \bar{Y} < 0.5) = P\left(\frac{-0.5-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}} < \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}} < \frac{0.5-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}}\right)$$
$$= P(0.97 < Z < 1.62) = 0.9306 - 0.8340 = 0.0966$$

(c) We have $\sigma_1 = \sigma_2 = 5$, $n = m = 21$ then

$$P\left(\frac{S_1^2}{S_2^2} < 2.124\right) = 1 - P\left(\frac{S_1^2}{S_2^2} > 2.124\right) = 1 - P\left(\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} > 2.124\frac{\sigma_2^2}{\sigma_1^2}\right) = 1 - P(F > 2.124)$$
$$= 1 - 0.05 = 0.95$$

where F is a random variable with F-distribution and $\nu_1 = n-1 = 20, \nu_2 = m-1 = 20$ degree of freedoms.

(d) We have $\mu_1 = 20.5$ and σ_1, is unknown $s_1^2 = 36$ and $n = 21$ then

$$P(\bar{X} > 22) = P\left(\sqrt{n}\frac{\bar{X}-\mu_1}{s_1} > \sqrt{n}\frac{22-\mu_1}{\sigma_1}\right)$$
$$= P(T > 2.528) = 0.01$$

where T is a random variable with t-student distribution and $\nu_1 = n-1 = 20$ degree of freedom.

Marking scheme for Q1:

1 point for each correct answer. Total - 4 points.

Q2. Assume that X^2 has chi-square distribution, find χ^2_α such that

(a) $P(X^2 > \chi^2_\alpha) = 0.8\,,$ with $\nu = 15$

(b) $P(X^2 < \chi^2_\alpha) = 0.975\,,$ with $\nu = 10$

(c) $P(15.119 < X^2 < \chi^2_\alpha) = 0.2\,,$ with $\nu = 13$

Solution to Q2:

(a) 10.307

(b) $P(X^2 > \chi^2_\alpha) = 0.025\,,$ thus $\chi^2_\alpha = 20.483\,.$

(c) $0.2 = P(15.119 < X^2 < \chi^2_\alpha) = P(X^2 < \chi^2_\alpha) - P(X^2 < 15.119)$ Since $P(X^2 > 15.119) = 0.3$ then $P(X^2 < 15.119) = 0.7$ and therefore, $P(X^2 < \chi^2_\alpha) = 0.2 + 0.7 = 0.9$ and $P(X^2 > \chi^2_\alpha) = 0.1$ thus $\chi^2_\alpha = 19.812\,.$

Marking scheme for Q2:

1 point for each correct answer. Total - 3 points.

Q3. Assume that T has t-student distribution,

(a) obtain $P(T > 3.833)\,,$ with $\nu = 8$

(b) obtain $P(0.559 < T < 6.869)\,,$ with $\nu = 5$

(c) obtain $t_{0.025}\,,$ with $\nu = 6$

(d) obtain $-t_{0.9}\,,$ with $\nu = 9$

(e) obtain $t_\alpha\,,$ such that $P(0.876 < T < t_\alpha) = 0.1$ with $\nu = 11$

Solution to Q3:

(a) 0.0025

(b) $P(0.559 < T < 6.869) = P(T < 6.869) - P(T < 0.559) = 1 - 0.0005 - (1 - 0.3) = 0.2995\,,$ thus

(c) 2.447

(d) $-t_{0.9} = t_{0.1} = 1.383$

(e) $0.1 = P(0.876 < T < t_\alpha) = P(T < t_\alpha) - P(T < 0.876) = P(T < t_\alpha) - (1 - 0.2)$ thus $P(T < t_\alpha) = 0.9$ and $P(T > t_\alpha) = 0.1$ thus $t_{0.1} = 1.363\,.$

Marking scheme for Q3:

1 point for each correct answer. Total - 4 points.

Q4. For an F-distribution, find

(a) $f_{0.05}$ with $\nu_1 = 5$ and $\nu_2 = 7\,.$

(b) $f_{0.99}$ with $\nu_1 = 5$ and $\nu_2 = 7\,.$

Solution to Q4:

(a) 3.972

(b) Since $f_{1-\alpha}(\nu_1, \nu_2) = \dfrac{1}{f_\alpha(\nu_2, \nu_1)}\,,$ we have $f_{0.99} = 1/10.455$

Marking scheme for Q4:

This question is not marked.

Q5. Discuss normality of the following data set:

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170,295,200,165,140,190,195,142,138,148,110,140,103,176,125,126,204,196,98,123,124
152,177,168,175,186,140,147,174,155,195
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Solution to Q5:

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Data=c(170,295,200,165,140,190,195,142,138,148,110,140,103,176,125,126,204,196,
98,123,124,152,177,168,175,186,140,147,174,155,195);
par(mfrow=c(1,2)); boxplot(Data);hist(Data,breaks=10)
```

Data do not seem to be symmetric. There is an outlier. They do not seem to be normal. After illumination of the outlying observation, we can find some slight detour from normality assumptions but Shapiro-Wilk (a test to examine a data set satisfying normality assumption) ensure that data set has normal distribution after removing the outlier.

Marking scheme for Q5:

This question is not marked.

Q6. (Bonus question - this question is not mandatory). Using R, illustrate the central limit theorem by generating $M = 300$ samples of size $n = 30$ from:

- a normal random variable with mean 5 and variance 1.5;
- a binomial random variable with 4 trials and probability of success 0.2.

Repeat the same procedure for samples of size $n = 200$. What do you observe? Hint: In each case, assess the normality using a histogram and a Q-Q plot.

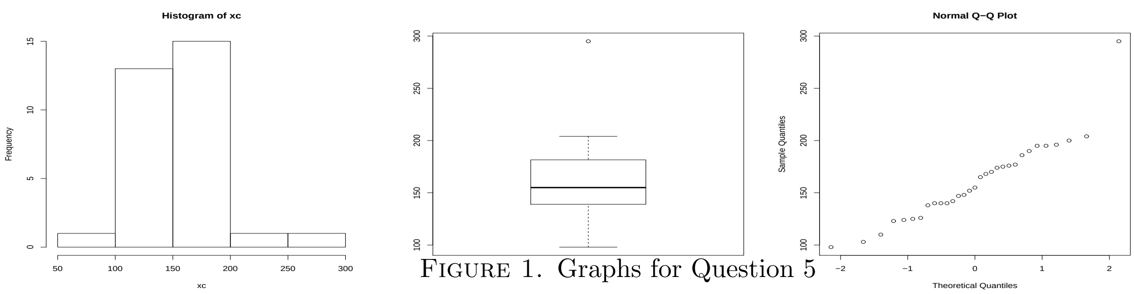

FIGURE 1. Graphs for Question 5