Assignment IV
Due date: 21 November 2018
Note: You have to drop off your assignment in the class. It is advised that you make a copy of your assignment.
Note: I will post solutions to all questions. However, only selected questions will be marked.
Note: Assignments after the due time will not be accepted.
Note: The electronic version of your assignment will not be accepted.

Q1. To estimate the mean of a population, the random sample X_1, X_2, X_3, X_4, X_5 are taken from the population with mean μ and variance σ^2 and the following estimators for mean are proposed: $\hat{\mu}_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$, $\hat{\mu}_2 = \frac{X_1 + X_2 + X_3}{3}$, $\hat{\mu}_3 = \frac{X_1 + X_2 + 3X_5}{5}$, $\hat{\mu}_4 = \frac{X_1 + X_2 + 2X_3}{5}$.

(a) Which one of the estimators are unbiased estimators?

(b) Obtain the variance of each estimator.

(c) Which estimator is the most efficient estimator?

Solution to Q1:

We have

(a) Since $E(\hat{\mu}_1) = \frac{1}{5}(E(X_1) + \ldots + E(X_5)) = \mu$, $E(\hat{\mu}_2) = \frac{1}{3}(E(X_1) + \ldots + E(X_3)) = \mu$, $E(\hat{\mu}_3) = \frac{1}{5}(E(X_1) + E(X_2) + 3E(X_5)) = \mu$, $E(\hat{\mu}_4) = \frac{1}{5}(E(X_1) + E(X_2) + 2E(X_3)) = \frac{4}{4}\mu$, $\hat{\mu}_1, \hat{\mu}_2$, and $\hat{\mu}_3$ are unbiased estimator and $\hat{\mu}_4$ is biased estimator.

(b) $\sigma^2_{\hat{\mu}_1} = \frac{1}{5^2}(\sigma^2_{X_1} + \ldots + \sigma^2_{X_5}) = \sigma^2/5$, $\sigma^2_{\hat{\mu}_2} = \frac{1}{5^2}(\sigma^2_{X_1} + \sigma^2_{X_1} + \sigma^2_{X_3}) = \sigma^2/3$, $\sigma^2_{\hat{\mu}_3} = \frac{1}{5^2}(\sigma^2_{X_1} + \sigma^2_{X_1} + 3^2\sigma^2_{X_5}) = \frac{11}{25}\sigma^2$, $\sigma^2_{\hat{\mu}_4} = \frac{1}{5^2}(\sigma^2_{X_1} + \sigma^2_{X_1} + 2^2\sigma^2_{X_3}) = \frac{6}{25}\sigma^2$,

(c) Since $\hat{\mu}_1$ has the smallest variance it is the most efficient estimator.

Marking scheme for Q1:

1 point for each correct answer of the first part, 1 point for each correct answer of the second part, 1 point for the correct answer to the third part. Total - 9 points.

Q2. Past experience indicates that the breaking strength of yarn used in manufacturing drapery material is normally distributed and that $\sigma = 2$ psi. A random sample of 15 specimens is tested and the average breaking strength is found to be $\bar{x} = 97.5$ psi.

(a) Find a 95% confidence interval on the true mean breaking strength.

(b) Find a 99% confidence interval on the true mean breaking strength.

Solution to Q2:

Conditions: normal population with known σ.
A 95% confidence interval is

$$\bar{x} \pm z_{.025}\frac{\sigma}{\sqrt{n}} = 97.5 \pm 1.96\left(\frac{2}{\sqrt{15}}\right) = 97.5 \pm 1.012140 = [96.48786, 98.51214].$$

A 99% confidence interval is

$$\bar{x} \pm z_{.005}\frac{\sigma}{\sqrt{n}} = 97.5 \pm 2.576\left(\frac{2}{\sqrt{15}}\right) = 97.5 \pm 1.330241 = [96.16976, 98.83024].$$

Marking scheme for Q2:

1 point for the correct use of confidence interval (i.e. $\overline{x} \pm z_{.025}\dfrac{\sigma}{\sqrt{n}}$ in the first part), 1 point for the correct answer in each part. Total - 4 points.

Q3. Two different brands of latex paint are being considered for use. Twenty specimens of each type of paint were selected, and the drying times, in hours, were as follows:

Paint A					Paint B				
3.5	2.7	3.9	4.2	3.6	4.7	3.9	4.5	5.5	4.0
2.7	3.3	5.2	4.2	2.9	5.3	4.3	6.0	5.2	3.7
4.4	5.2	4.0	4.1	3.4	5.5	6.2	5.1	5.4	4.8
3.4	4.3	3.2	4.2	2.8	6.5	4.9	6.2	6.8	5.8

(a) Find a 98% confidence interval for μ_A and μ_B , where μ_B and μ_A are the mean drying times.

(b) Find a 95% confidence interval for σ_A^2 and σ_B^2 , where σ_A^2 and σ_A^2 are the variance of drying times.

(c) Assume the drying time is normally distributed with $\sigma_A = \sigma_B$. Find a 95% confidence interval on $\mu_B - \mu_A$, where μ_B and μ_A, are the mean drying times.

Solution to Q3:

(a) We have $1 - \alpha = 0.98$ therefore, $t_{\alpha/2} = 2.539$ with $\nu = 19$, $\bar{X}_A = 3.76$, $\bar{X}_B = 5.215$, $S_A = 0.740128$, $S_B = 0.8785604$, and $n = m = 20$, and the confidence interval for μ_A is

$$\bar{X}_A \pm t_{\alpha/2} S_A/\sqrt{n} = 3.76 \pm 2.539 \times 0.740128/\sqrt{20} = (3.339801, 4.180199)\,.$$

The confidence interval for μ_A is

$$\bar{X}_B \pm t_{\alpha/2} S_B/\sqrt{m} = 5.215 \pm 2.539 \times 0.8785604/\sqrt{20} = (4.716208, 5.713792)\,.$$

(b) We have $1 - \alpha = 0.95$ and $\chi^2_{\alpha/2} = 32.852$ and $\chi^2_{1-\alpha/2} = 8.907$ with $\nu = 19$. The confidence interval for σ_A^2 is

$$\left(\frac{(n-1)S_A^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S_A^2}{\chi^2_{1-\alpha/2}}\right) = (0.3168148, 1.168519)\,.$$

The confidence interval for σ_B^2 is

$$\left(\frac{(m-1)S_B^2}{\chi^2_{\alpha/2}}, \frac{(m-1)S_B^2}{\chi^2_{1-\alpha/2}}\right) = (0.4464112, 1.646514)\,.$$

(c) $SP = \dfrac{(n-1)S_A^2 + (m-1)S_B^2}{n+m-2} = 0.6598289$ and $1 - \alpha = 0.95$ therefore, $t_{\alpha/2} \simeq 2.021$ with $\nu = 38$, the 95% confidence interval on $\mu_B - \mu_A$ is

$$\bar{X}_B - \bar{X}_A \pm t_{\alpha/2}\sqrt{SP}\sqrt{1/n + 1/m} = (0.935863, 1.974137)\,.$$

Marking scheme for Q3:

1 point for the correct use of confidence interval (i.e. $\bar{X}_A \pm t_{\alpha/2} S_A/\sqrt{n}$ in the first part, $\left(\dfrac{(n-1)S_A^2}{\chi^2_{\alpha/2}}, \dfrac{(n-1)S_A^2}{\chi^2_{1-\alpha/2}}\right)$ in the second part, and $\bar{X}_A - \bar{X}_B \pm t_{\alpha/2}\sqrt{SP}\sqrt{1/n + 1/m}$ in the third parts), 1 point for the correct answer in each part. Total - 8 points.

Q4. The diameter holes for a cable harness follow a normal distribution with $\sigma = 0.01$ pounce. For sample of size 10, an average diameter is 1.5045 inch.

(a) Find a 99% confidence interval on the mean hole diameter.

(b) Repeat this for $n = 100$.

Solution to Q4:

Conditions: normal population with known σ.
A 99% confidence interval for $n = 10$ is

$$\overline{x} \pm z_{.005}\frac{\sigma}{\sqrt{n}} = 1.5045 \pm 1.96\left(\frac{0.01}{\sqrt{10}}\right) = 1.5045 \pm 0.008146027 = [1.496354, 1.512646].$$

A 99% confidence interval for $n = 100$ is

$$\overline{x} \pm z_{.005}\frac{\sigma}{\sqrt{n}} = 1.5045 \pm 1.96\left(\frac{0.01}{\sqrt{100}}\right) = 1.5045 \pm 0.002576 = [1.501924, 1.507076].$$

Marking scheme for Q4:

This question is not marked.