Assignment V
Due date: 3 December 2018
Note: You have to drop off your assignment in the class. It is advised that you make a copy of your assignment.
Note: I will post solutions to all questions. However, only selected questions will be marked.
Note: Assignments after the due time will not be accepted.
Note: The electronic version of your assignment will not be accepted.

Q1. The brightness of television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. An engineer thinks that one has to use 300 microamps of current to achieve the required brightness level. A sample of size $n = 20$ has been taken to verify the engineer's hypotheses.

(a) Formulate the null and the alternative hypotheses. Use two-sided test alternative.

(b) For the sample of size $n = 20$ we obtain $\overline{x} = 319.2$ and $s = 18.6$. Test the hypotheses from part (a) with $\alpha = 2\%$.

(c) Test the hypothesis using P-value when the significant level is $\alpha = 0.01$.

Solution to Q1:

(a) We want to verify $\mu = 300$, thus we test $H_0 : \mu = 300$ against $H_1 : \mu \neq 300$.

(b) Critical region is $t_0 = \dfrac{\overline{x} - 300}{s/\sqrt{n}} > t_{\alpha/2}$ or $t_0 = \dfrac{\overline{x} - 300}{s/\sqrt{n}} < -t_{\alpha/2}$. We have $t_{\alpha/2} = 2.539$, where $\alpha = 0.02$ and degrees of freedom is 19.

$$t_0 = \frac{\overline{x} - 300}{s/\sqrt{n}} = \frac{319.2 - 300}{18.6/\sqrt{20}} = 4.61.$$

Since the test statistics is in the critical region, we reject H_0 .

(c) p-value:

$$2\,\min(P(T > t_0), P(T < t_0)) = 2\,\min(P(T > 4.61), 1 - P(T < 4.61))\,.$$

From the table you can find out that $P(t_{19} > 4.61) < 0.0005$ so that $2\,\min(P(T > 4.61), 1 - P(T < 4.61)) = 2\,P(T > 4.61)$ and p-value$< 2(0.0005) = 0.001$. Reject H_0.

Marking scheme for Q1:

Total - 4 points. 1 point for part a), 2 points for part b); 1 point for part c)

Q2. Two different brands of latex paint are being considered for use. Twenty specimens of each type of paint were selected, and the drying times, in hours, were as follows:

Paint A					Paint B				
3.5	2.7	3.9	4.2	3.6	4.7	3.9	4.5	5.5	4.0
2.7	3.3	5.2	4.2	2.9	5.3	4.3	6.0	5.2	3.7
4.4	5.2	4.0	4.1	3.4	5.5	6.2	5.1	5.4	4.8
3.4	4.3	3.2	4.2	2.8	6.5	4.9	6.2	6.8	5.8

Let μ_B and μ_A be the mean drying times of Paint A and Paint B, respectively and σ_A^2 and σ_A^2 are the corresponding variances of drying times. Moreover, assume that σ_A^2 and σ_A^2 are unknown but equal.

(a) Test the hypothesis $H_0 : \mu_A = 4$ versus $H_1 : \mu_A > 4$, based on the critical region and P-value, if the significant level is $\alpha = 0.05$.

(b) Test the hypothesis $H_0 : \mu_B = 5$ versus $H_1 : \mu_B \neq 5$, based on the critical region and P-value, if the significant level is $\alpha = 0.02$.

(c) Test the hypothesis $H_0 : \mu_A = \mu_B$ versus $H_1 : \mu_A < \mu_B$, based on the critical region and P-value, if the significant level is $\alpha = 0.01$.

(d) Test the hypothesis $H_0 : \mu_A = \mu_B$ versus $H_1 : \mu_A \neq \mu_B$, based on the critical region and P-value, if the significant level is $\alpha = 0.02$.

Solution to Q2:

We have $\bar{X}_A = 3.76$, $\bar{X}_B = 5.215$, $S_A = 0.740128$, $S_B = 0.8785604$, and $n = m = 20$.

(a) The critical region is $\dfrac{\bar{X}_A - 4}{S_A/\sqrt{n}} > t_\alpha$, with $t_\alpha = t_{0.05} = 1.729$ with $\nu = 19$ degrees of freedom. Since $\dfrac{\bar{X}_A - 4}{S_A/\sqrt{n}} = -1.959352$, the test statistics is not in the critical region and therefore we do not have sufficient evidence to reject H_0.
$P - value$ equals $P(T > \dfrac{\bar{X}_A - 4}{S_A/\sqrt{n}}) = P(T > -1.959352) = P(T < 1.959352) = 1 - P(T > 1.959352)$, by Table A.4 $0.10 < P(T > 1.959352) < 0.15$ and therefore $0.85 < 1 - P(T > 1.959352) < 0.90$, therefore P-value is bigger than significant level $\alpha = 0.05$, and we do not have sufficient evidence to reject H_0.

(b) The critical region is $\dfrac{\bar{X}_B - 5}{S_B/\sqrt{m}} > t_{\alpha/2}$, or $\dfrac{\bar{X}_B - 5}{S_B/\sqrt{m}} < -t_{\alpha/2}$, with $t_{\alpha/2} = t_{0.01} = 2.539$ with $\nu = 19$ degrees of freedom. Since $\dfrac{\bar{X}_B - 5}{S_B/\sqrt{m}} = 1.245691$, the test statistics is not in the critical region and therefore we do not have sufficient evidence to reject H_0.
$P - value$ equals $2\min(P(T > \dfrac{\bar{X}_B - 5}{S_B/\sqrt{m}}), P(T < \dfrac{\bar{X}_B - 5}{S_B/\sqrt{m}})) = 2P(T > 1.24569)$, by Table A.4 $0.10 < P(T > 1.24569) < 0.15$ and therefore $0.20 < 2P(T > 1.24569) < 0.30$, therefore P-value is bigger than significant level $\alpha = 0.02$, and we do not have sufficient evidence to reject H_0.

(c) The critical region is $\dfrac{\bar{X}_A - \bar{X}_B}{SP\sqrt{1/m + 1/n}} < -t_\alpha$, with $t_\alpha = t_{0.01} \simeq 2.423$ with $\nu = 38$ degrees of freedom. Since $\dfrac{\bar{X}_A - \bar{X}_B}{SP\sqrt{1/m + 1/n}} = -5.73836$, the test statistics is in the critical region and therefore we reject H_0.
$P - value$ equals $2\min(P(T > -5.73836), P(T < -5.73836)) = 2P(T < -5.73836) = 2P(T > -5.73836) \simeq 0$, by Table A.4 and therefore P-value is less than significant level $\alpha = 0.01$, and we reject H_0.

(d) The critical region is $\dfrac{\bar{X}_A - \bar{X}_B}{SP\sqrt{1/m + 1/n}} > t_{\alpha/2}$, or $\dfrac{\bar{X}_A - \bar{X}_B}{SP\sqrt{1/m + 1/n}} < -t_{\alpha/2}$, with $t_{\alpha/2} = t_{0.01} = 2.423$ with $\nu = 38$ degrees of freedom. Since $\dfrac{\bar{X}_A - \bar{X}_B}{SP\sqrt{1/m + 1/n}} = -5.73836$, the test statistics is in the critical region and therefore we reject H_0.
$P - value$ equals $2\min(P(T > -5.73836), P(T < -5.73836)) = 2P(T < -5.73836) = 2P(T > 5.73836) \simeq 0$, by Table A.4 and therefore P-value is less than significant level $\alpha = 0.02$, and we reject H_0.

Marking scheme for Q2:

In each part 1 point for critical region, 1 point for correct inference, and 1 point for correct p-value. Total - 12 points.

Q3. A soft-drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of μ milliliters and an unknown variance σ^2 squared milliliters. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If $\bar{X}$ falls in the interval $200.3493 < \bar{X} < 204.3333$, the machine is thought to be operating satisfactorily; otherwise, we conclude that $\mu \neq 200$ milliliters. If the sample variance $S^2 = 16$

(a) Find the probability of committing a type I error when $\mu = 200$ milliliters.

(b) Find the probability of committing a type II error when $\mu = 215$ milliliters.

Solution to Q3:

(a) When $\mu = 200$, $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ has Student t-distribution with $n-1$ degrees of freedom. Moreover,

$$\text{Type I error} = P(\text{rejection } H_0 \text{when it is true}) = 1 - P(200.3493 < \bar{X} < 204.3333)$$
$$= 1 - P(0.262 < T < 3.250) = 1 - (P(T > 0.262) - P(T > 3.355)) = 1 - (0.40 - 0.005) = 0.605\,.$$

(b) When $\mu = 215$, $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ has Student t-distribution with $n-1$ degrees of freedom. Moreover,

$$\text{Type II error} = P(non\text{rejection } H_0 \text{when it is false}) = P(200.3493 < \bar{X} < 204.3333)$$
$$= P(-10.98803 < T < -8.000025) = P(T < -8.000025) - P(T < -10.98803) \simeq 0\,.$$

Marking scheme for Q3:

Each part has 1 point. Total - 2 points.

Q4. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 49 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 36 bulbs has an average life of 788 hours. Use a P-value in your answer with significant level $\alpha = 0.05$.

Solution to Q4:

We have $\sigma = \sqrt{49} = 7$, $\bar{x} = 788$, and $n = 30$. The critical region is $\dfrac{\bar{X} - 800}{\sigma/\sqrt{n}} > z_{\alpha/2}$, or $\dfrac{\bar{X} - 800}{\sigma/\sqrt{n}} < -z_{\alpha/2}$, with $z_{\alpha/2} = z_{0.025} = 1.96$. Since $\dfrac{\bar{X} - 800}{\sigma/\sqrt{n}} = 10.28571$, the test statistics is in the critical region and therefore we reject H_0.

$P - value$ equals $2\min(P(Z > 10.28571), P(Z < 10.28571)) = 2P(Z >< 10.28571) \simeq 0$, by Table A.3 and therefore P-value is smaller than significant level $\alpha = 0.05$, and we reject H_0.

Marking scheme for Q4:

Each part has 1 point. Total - 2 points.