TELFER

VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

ADM 2304 – Final Examination (Winter 2016)

APPLIED STATISTICAL METHODS IN BUSINESS

PRINT YOUR NAME: ____

STUDENT NUMBER: ______________________________

SECTION (please circle one): M N P Q R S

Exam Length: **9 pages (including cover page) and 3 hours**

Appendices: **8 pages of Minitab output and 4 pages of statistical tables**

Complete all tests with hypotheses, test statistic, p-value or critical value, decision, and conclusion. Use the 5% significance level unless otherwise indicated. Explain your answers when requested.

Calculators and one sheet of notes (8.5 x 14 in.) are allowed.

Please hand back all exam materials (exam booklet, appendices and tables), but keep your personal sheet of notes for future reference.

Question	Value	Mark
1	12	
2	13	
3	8	
4	18	
5	24	
Total	**75**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ____

Question 1. [12 marks]

Appendix A contains data on the proportion of high school graduates and the proportion of college graduates for a sample of neighbourhoods. You are interested in determining if the proportion of high school graduates is more than 6% higher than the proportion of college graduates in the population of Toronto neighbourhoods.

(a) Is the appropriate test based on paired or independent samples? Explain briefly.

[1]

(b) Should your test be a parametric or a non-parametric test? Explain briefly with specific reference to the appropriate boxplot(s).

[2]

(c) Perform the appropriate test at the 5% level of significance. Use the critical value approach.

[4]

(d) If you performed a paired test in (c), then perform the independent samples test now. If you performed an independent samples test in (c), now perform a paired test.

[4]

(e) Why do the two tests in parts (c) and (d) result in different conclusions?

[1]

Question 2. [13 marks]

A random sample of 51 neighbourhoods was selected from the population of neighbourhoods where the proportion of university graduates was less than 25% and a random sample of 59 neighbourhoods was taken from the population of neighbourhoods where the proportion of university graduates was more than 25%.

In the first sample of 51 neighbourhoods, 27 had median incomes over $25,000; in the second sample of 59 neighbourhoods, 49 had median incomes over $25,000.

(a) Test whether the proportions of neighbourhoods with median incomes over $25,000 are different in the two populations. Calculate a z-statistic and complete the test using the p-value approach and a 1% level of significance.

[4]

(b) Calculate a 99% confidence interval to estimate the difference in the two population proportions. Explain how this interval leads to the same conclusion as above.

[2]

(c) Fill in the blanks:
The test and confidence interval above assume that the ___________________________________

[3] ___________________________________ have a ______________________________ distribution.
We are assured this is true because

____.

(d) Now do the same test using a chi-square test of homogeneity. Use the p-value approach and a 1% level of significance.

[4]

Question 3. [8 marks]

On April 19, 2016, Manitoba held its provincial election. The morning after the vote, the breakdown of the vote was: Progressive Conservative 53.7%, NDP 25.2%, Liberal 14.4%, other 6.7%.

As of April 12, the latest poll showed a sample breakdown of 51.3% PCs, 25.9% NDP, 18.2% Liberal, and 4.6% other.

(a) Assuming a sample size of 1000 for the poll, test whether the sample breakdown could be considered an accurate reflection of the distribution of the actual vote one week later. Use a 5% level of significance and the critical value approach.

[4]

(b) Now suppose that in a sample of 25 university students in St. Boniface, Manitoba, only one voted for the Liberal party. Test whether this constitutes sufficient evidence to show that the proportion of university students in St. Boniface who voted for the Liberal party is less than 14.4%.

[4]

Question 4. [18 marks]

An industrial process engineer conducted an experiment to determine how the **machines** used and the **shift** in which the machines are operated affect the **processing time** in minutes. The details of the experiment are given in Appendix B.

(a) Here is part of the analysis given as an output table. Complete the missing entries in this table. Show your calculations for MSE, R-Sq, and 's'.

Analysis of Variance for ProcessTime

Source	DF	SS	MS
Shift	__	______	____
Machine	__	7.502	____
Machine*Shift	__	97.662	____
Error	__	119.547	____
Total	26	384.560	

S = ______ R-Sq = ______ R-Sq(adj) = 55.10%

[4]

(b) Now show an alternative method of calculating the MSE which combines the standard deviations in the appendix (do not complete the calculation, just show how they are combined).

[1]

(c) What do you observe about the possible effects of Shift and Machine (separately and jointly) on the average processing time? Justify your answer with clear references to the relevant plot. Which machine results in the same average processing time, regardless of the shift?

[3]

(d) Test whether the effect of the Shift on the processing time depends on the Machine. Use a 5% level of significance.

[3]

(e) If one were to do a test to compare the mean processing times for the three shifts and the p-value was found to be 0.000, what would you conclude? Given the result in part (d) above, would you consider this conclusion meaningful? Explain why you would consider it meaningful or not.

[2]

(f) Using the Bonferroni method of multiple comparisons, calculate the margin of error for comparing the treatment means. What do you conclude about the mean process times for machine 3 operated in the morning and machine 2 operated in the night? Which is the best combination?

{Hint: You will need one of the following values for $t^* = t_{\alpha/(2m)}(df_E)$ where 'm' is the number of pair-wise comparisons. Based on the correct value of 'm', **choose the appropriate value** from this list: $t_{0.025} = 2.1009$, $t_{0.00833} = 2.6393$, $t_{0.00625} = 2.7745$, $t_{0.002778} = 3.1482$, $t_{0.001389} = 3.4622$, or $t_{0.000694} = 3.7745$}

[5]

Question 5. [24 marks]

Appendix C estimates several regression models of house sale prices. The variables in the dataset are:

Variable	Description
ID	1-522
Price	Sale price of house (thousands of dollars)
SqFt	Size of house in square feet
Bedrms	# of bedrooms
Bathrms	# of bathrooms
AC	1 if Air Conditioned; 0 if not
Garage	# of cars
Pool	1 if pool; 0 if no pool
Year	Year house was built
Quality	Construction quality: 1 for high, 2 for medium,3 for low
Style	Qualitative indicator of architectural style
Lotsize	Size of Lot in square feet
Highway	1 if house by highway, 0 if not

(Kutner et al, Applied Linear Statistical Models, 5th ed. McGraw-Hill, 2005)

Some of the values of these variables are shown for a few houses at the end of Appendix C.

(a) Examine the residual plot for Model 1. Discuss any problems with the validity of the linear model assumptions.

[2]

(b) Now examine the residual plot for Model 2 (the dependent variable here is the logarithm (base 10) of price). Have the problems above been resolved?

[2]

(c) Model 3 drops a number of observations from Model 2. What changes do you see from Model 2 to Model 3?

[2]

(d) Test at the 5% level of significance whether Model 3 is useful for prediction.
[3]

(e) Test at the 1% level of significance whether the **SqFt** variable is an important variable in Model 3. Include the hypotheses, test statistic, rejection region, decision, and conclusion.
[4]

(f) What does the regression coefficient of the **SqFt** variable estimate? Be as precise as possible.
[2]

(g) Explain whether multicollinearity is an issue that might affect the value of the coefficients.
[1]

(h) What is the interpretation of the coefficient of the **Pool** variable? Be as precise as possible.
[2]

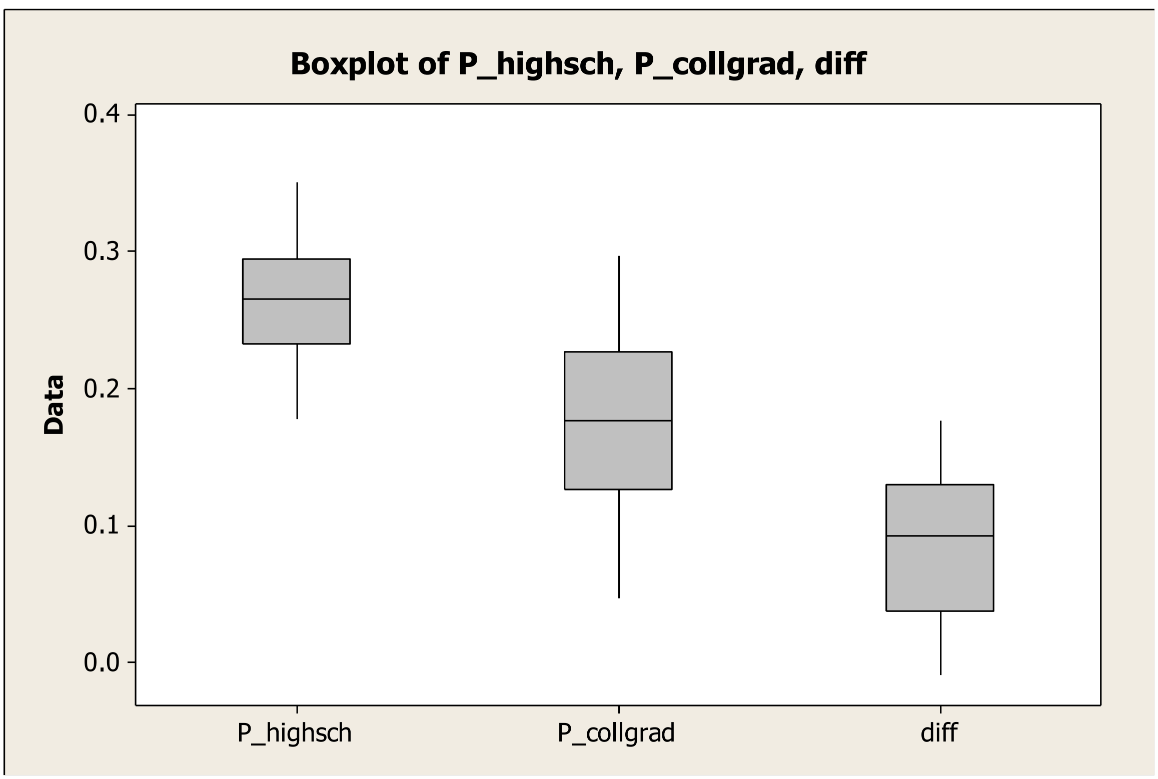
(i) Model 4 calculated 99% intervals for the logarithm (base 10) of the price of a specific house with the characteristics as shown under "Value of Predictors", but these intervals have been erased. Calculate the appropriate 99% interval for the price (in logarithmic terms) of a house with the given characteristics.
[3]

(j) What is the estimated mean price of houses with the same characteristics? Use the appropriate units of measurement. (If you cannot complete the calculation, then show how it would be calculated and give an approximate value.)
[1]

(k) According to the output from the Best Subsets procedure, which model might be considered the best? Answer using two different criteria (two equivalent criteria counts as the same criterion).
[2]

Appendix A

P_highsch	P_collgrad	diff	P_collgrad+0.06
0.327374	0.297207	0.030168	0.357207
0.234417	0.105691	0.128726	0.165691
0.325942	0.239468	0.086475	0.299468
0.192521	0.202216	-0.00969	0.262216
0.264205	0.181818	0.082386	0.241818
0.343381	0.235911	0.107471	0.295911
0.296987	0.131994	0.164993	0.191994
0.288227	0.247632	0.040595	0.307632
0.231750	0.206257	0.025492	0.266257
0.245310	0.212121	0.033189	0.272121
0.283186	0.176991	0.106195	0.236991
0.270014	0.232022	0.037992	0.292022
0.196835	0.158259	0.038576	0.218259
0.261248	0.124819	0.136430	0.184819
0.218891	0.122939	0.095952	0.182939
0.266075	0.177384	0.088692	0.237384
0.177778	0.046914	0.130864	0.106914
0.264822	0.166008	0.098814	0.226008
0.350900	0.173522	0.177378	0.233522
0.280488	0.109756	0.170732	0.169756



Two-Sample T-Test and CI: P_highsch, P_collgrad

```
Two-sample T for P_highsch vs P_collgrad

             N    Mean   StDev  SE Mean
P_highsch   20  0.2660  0.0489    0.011
P_collgrad  20  0.1774  0.0595    0.013


Difference = mu (P_highsch) - mu (P_collgrad)
Estimate for difference:  0.088571
95% lower bound for difference:  ________
T-Test of difference = ____ (vs ____): T-Value = ____  P-Value = ____  DF = 36
```

Paired T-Test and CI: P_highsch, P_collgrad

```
Paired T for P_highsch - P_collgrad

             N      Mean     StDev   SE Mean
P_highsch   20  0.266018  0.048851  0.010923
P_collgrad  20  0.177446  0.059501  0.013305
Difference  20  0.088571  0.053563  0.011977


95% lower bound for mean difference: _______
T-Test of difference = ____ (vs ____): T-Value = ____  P-Value = ____
```

Wilcoxon Signed Rank Test: diff

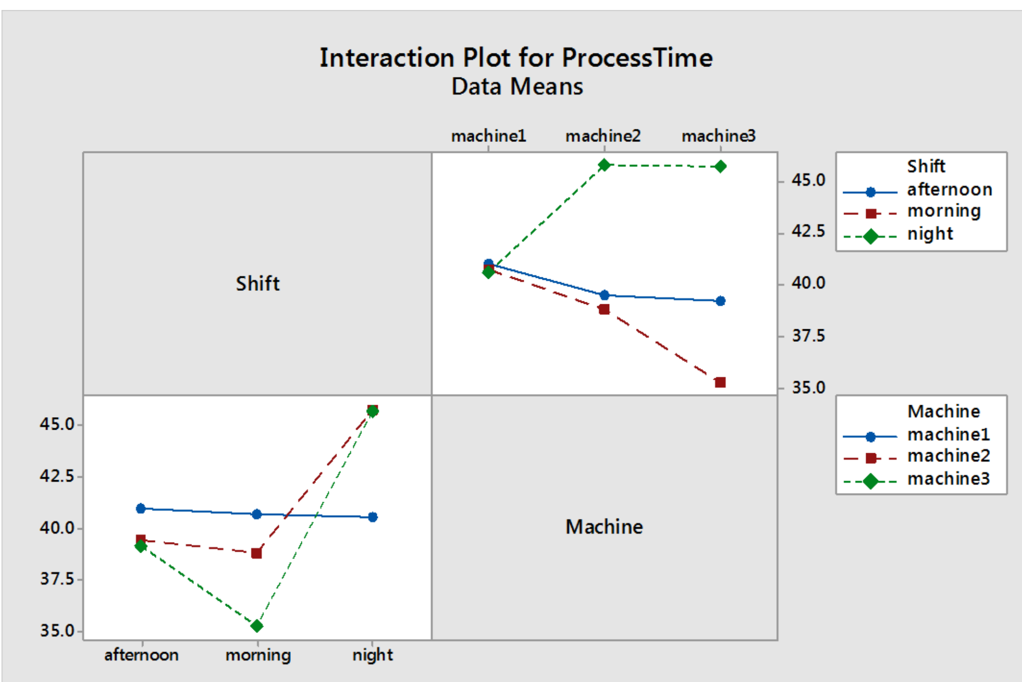
```
Test of median = 0.06000 versus median > 0.06000

             N
           for   Wilcoxon         Estimated
       N  Test  Statistic      P     Median
diff  20    20      166.0  0.012    0.08805
```

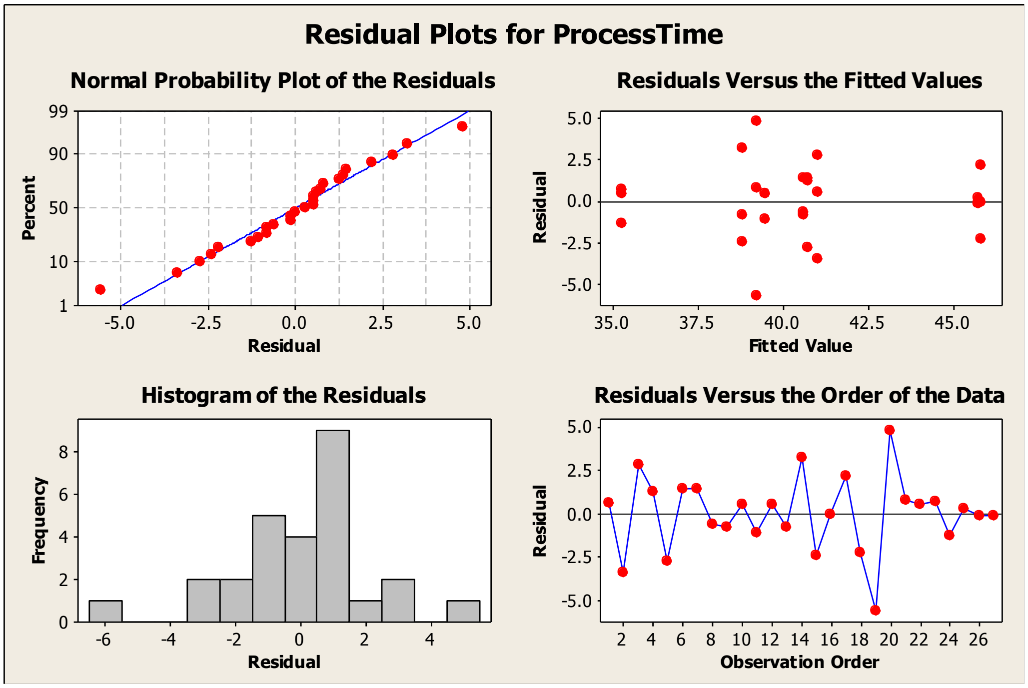
Mann-Whitney Test and CI: P_highsch, P_collgrad+0.06

```
                   N   Median
P_highsch         20  0.26545
P_collgrad+0.06   20  0.23719


Point estimate for ETA1-ETA2 is 0.02778
95.0 Percent CI for ETA1-ETA2 is (-0.00729,0.06236)
W = 464.0
Test of ETA1 = ETA2 vs ETA1 > ETA2 is significant at 0.0739
```

Appendix B

```
Rows: Shift  Columns: Machine

                  1       2       3     All

Afternoon     41.00   39.47   39.20   39.89
              3.143   0.924   5.246   3.205

               41.6    40.0    33.6
               37.6    38.4    44.0
               43.8    40.0    40.0

Morning       40.73   38.80   35.27   38.27
              2.369   2.884   1.102   3.090

               42.0    38.0    35.8
               38.0    42.0    36.0
               42.2    36.4    34.0

Night         40.60   45.80   45.73   44.04
              1.217   2.200   0.231   2.875

               42.0    45.8    46.0
               40.0    48.0    45.6
               39.8    43.6    45.6

All           40.78   41.36   40.07   40.73
              2.067   3.834   5.307   3.846

Cell Contents:  ProcessTime  :  Mean
                ProcessTime  :  Standard deviation
                ProcessTime  :  DATA
```





Appendix C

Model 1. Regression Analysis: Price versus SqFt, Bedrms, ...

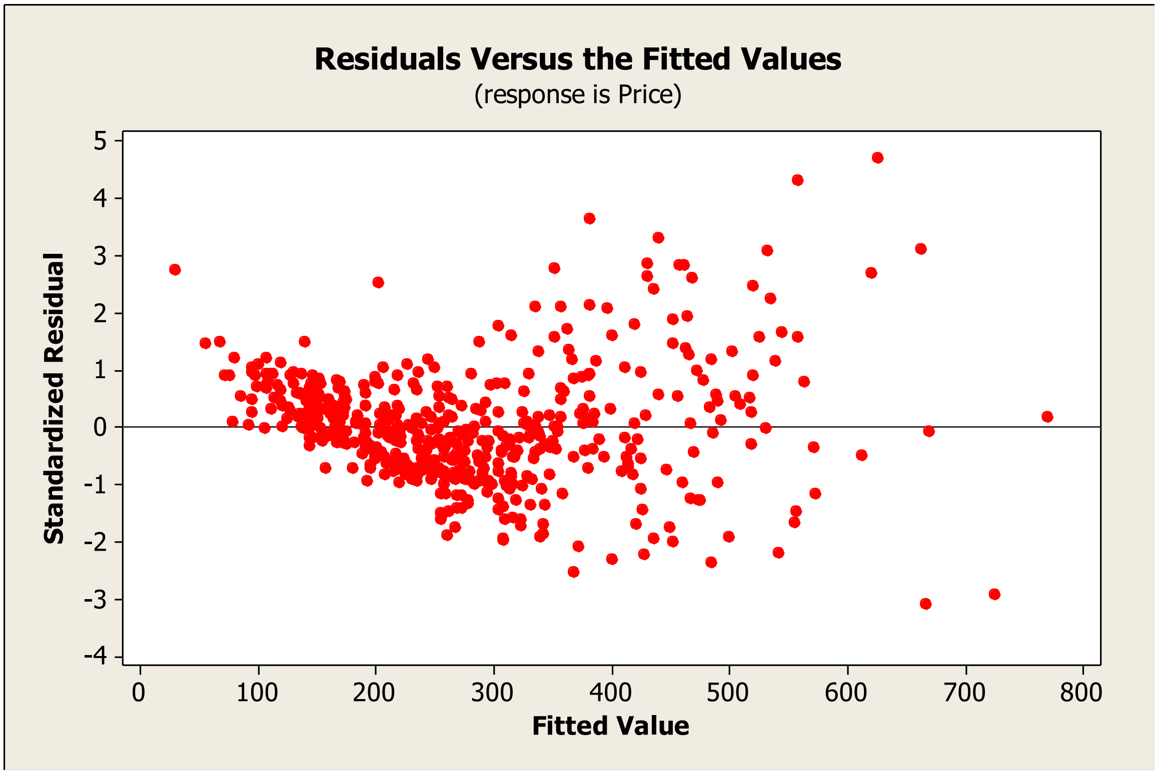
```
The regression equation is
Price = - 2394 + 0.130 SqFt - 8.99 Bedrms + 3.21 Bathrms - 13.5 AC
        + 13.9 GarageNoCars + 9.2 Pool + 1.27 Year - 48.2 Quality - 9.45 Style
        + 0.00119 LotSize - 38.3 Highway
```

Predictor	Coef	SE Coef	T	P
Constant	-2394.1	434.3	-5.51	0.000
SqFt	0.130344	0.007677	_____	_____
Bedrms	-8.986	3.517	-2.55	0.011
Bathrms	3.211	4.602	0.70	0.486
AC	-13.480	8.602	-1.57	0.118
GarageNoCars	13.942	5.463	2.55	0.011
Pool	9.24	11.26	0.82	0.412
Year	1.2660	0.2188	5.78	0.000
Quality	-48.224	7.408	-6.51	0.000
Style	-9.450	1.433	-6.59	0.000
LotSize	0.0011899	0.0002576	4.62	0.000
Highway	-38.30	19.64	-1.95	0.052

S = 63.5436   R-Sq = 79.2%   R-Sq(adj) = 78.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	11	7851637	713785	176.78	0.000
Residual Error	510	2059275	4038		
Total	521	9910912			



Model 2. Regression Analysis: log10Price versus SqFt, Bedrms, ...

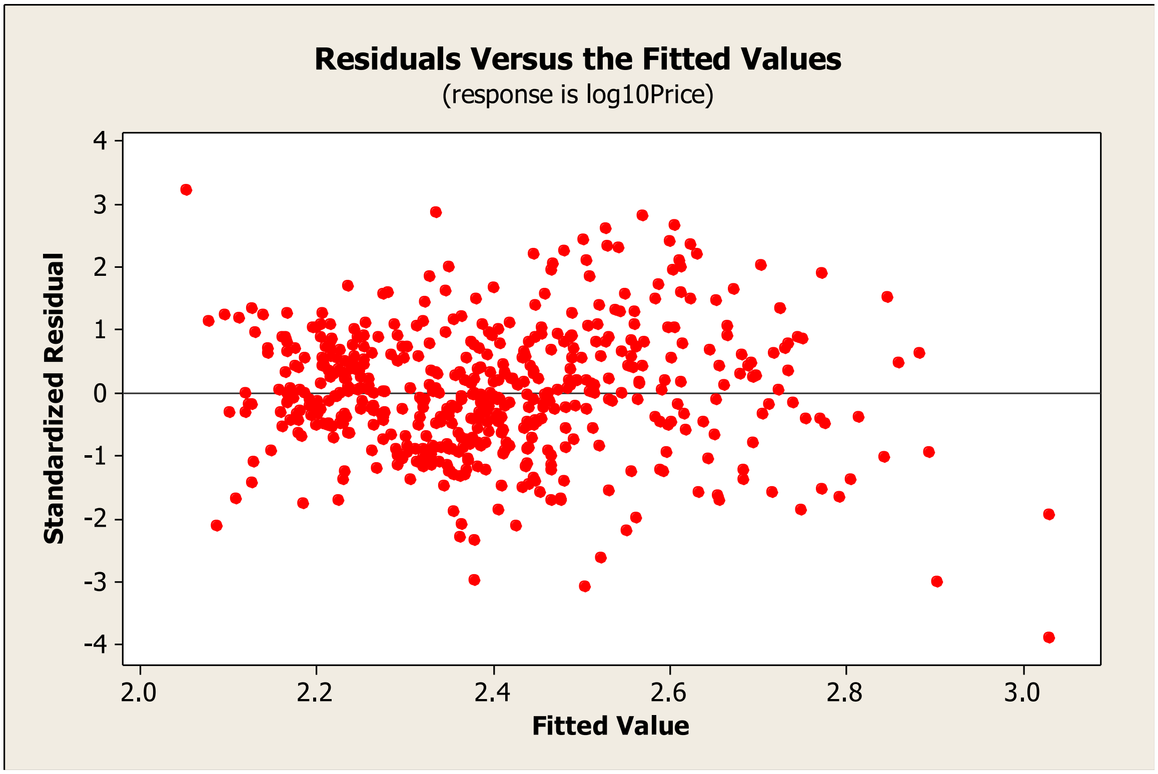
```
The regression equation is
log10Price = - 1.19 + 0.000145 SqFt - 0.00120 Bedrms + 0.0149 Bathrms
             + 0.0112 AC + 0.0175 GarageNoCars + 0.0218 Pool + 0.00168 Year
             - 0.0658 Quality - 0.00747 Style + 0.000002 LotSize
             - 0.0389 Highway
```

Predictor	Coef	SE Coef	T	P
Constant	-1.1921	0.5322	-2.24	0.026
SqFt	0.00014518	0.00000941	______	______
Bedrms	-0.001201	0.004311	-0.28	0.781
Bathrms	0.014857	0.005640	2.63	0.009
AC	0.01119	0.01054	1.06	0.289
GarageNoCars	0.017541	0.006695	2.62	0.009
Pool	0.02182	0.01380	1.58	0.114
Year	0.0016781	0.0002682	6.26	0.000
Quality	-0.065833	0.009079	-7.25	0.000
Style	-0.007470	0.001757	-4.25	0.000
LotSize	0.00000207	0.00000032	6.54	0.000
Highway	-0.03891	0.02407	-1.62	0.107

S = 0.0778770   R-Sq = 83.1%   R-Sq(adj) = 82.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	11	15.2179	1.3834	228.11	0.000
Residual Error	510	3.0931	0.0061		
Total	521	18.3110			



Model 3. Regression Analysis: log10Price* versus SqFt, Bedrms, ...

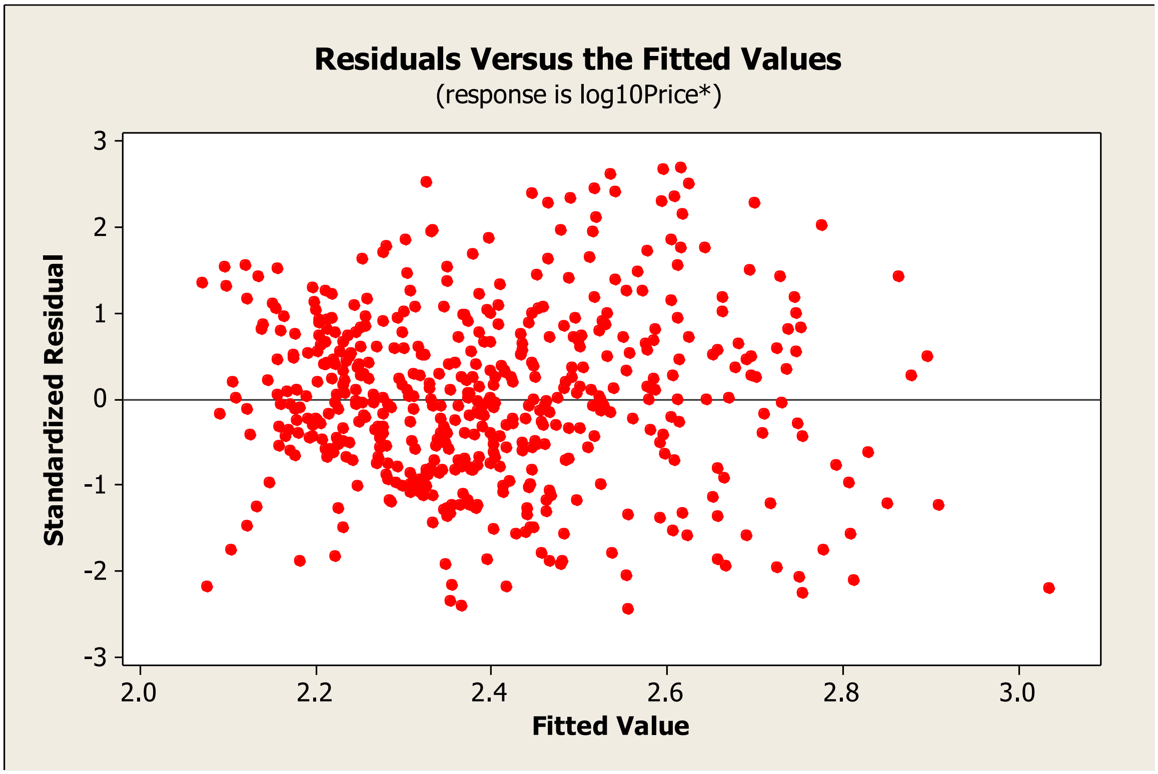
```
The regression equation is
log10Price* = - 1.79 + 0.000155 SqFt + 0.00071 Bedrms + 0.0206 Bathrms
              + 0.00696 AC + 0.0142 GarageNoCars + 0.0361 Pool + 0.00195 Year
              - 0.0502 Quality - 0.00855 Style + 0.000002 LotSize
              - 0.0374 Highway

512 cases used, 10 cases contain missing values
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	-1.7946	0.4980	-3.60	0.000	
SqFt	0.00015509	0.00000887	_____	______	3.8
Bedrms	0.000713	0.004083	0.17	0.861	1.6
Bathrms	0.020636	0.005460	3.78	0.000	3.3
AC	0.006962	0.009764	0.71	0.476	1.3
GarageNoCars	0.014177	0.006145	2.31	0.021	1.6
Pool	0.03609	0.01295	2.79	0.006	1.1
Year	0.0019519	0.0002508	7.78	0.000	1.9
Quality	-0.050165	0.008598	-5.83	0.000	3.0
Style	-0.008549	0.001624	-5.26	0.000	1.7
LotSize	0.00000202	0.00000030	6.69	0.000	1.2
Highway	-0.03738	0.02202	-1.70	0.090	1.0

S = 0.0711666   R-Sq = 85.6%   R-Sq(adj) = 85.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	11	15.0491	1.3681	270.12	0.000
Residual Error	500	2.5323	0.0051		
Total	511	17.5814			



Model 4. Regression Analysis: log10Price* versus SqFt, Bathrms, ...

```
The regression equation is
log10Price* = - 1.85 + 0.000155 SqFt + 0.0212 Bathrms + 0.0147 GarageNoCars
              + 0.0367 Pool + 0.00198 Year - 0.0510 Quality - 0.00856 Style
              + 0.000002 LotSize - 0.0379 Highway

512 cases used, 10 cases contain missing values
```

Predictor	Coef	SE Coef	T	P
Constant	-1.8508	0.4907	-3.77	0.000
SqFt	0.00015500	0.00000869	______	______
Bathrms	0.021161	0.005207	4.06	0.000
GarageNoCars	0.014665	0.006102	2.40	0.017
Pool	0.03670	0.01290	2.84	0.005
Year	0.0019848	0.0002462	8.06	0.000
Quality	-0.050956	0.008453	-6.03	0.000
Style	-0.008562	0.001621	-5.28	0.000
LotSize	0.00000199	0.00000030	6.67	0.000
Highway	-0.03793	0.02197	-1.73	0.085

S = 0.0710656   R-Sq = 85.6%   R-Sq(adj) = 85.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	9	15.0462	1.6718	331.03	0.000
Residual Error	502	2.5353	0.0051		
Total	511	17.5814			

Predicted Values for New Observations

```
New
Obs      Fit   SE Fit         99% CI                  99% PI
  1  2.91730  0.01928  (_______, _______)  (_______, _______)X

X denotes a point that is an outlier in the predictors.

Values of Predictors for New Observations

New
Obs  SqFt  Bathrms  GarageNoCars       Pool  Year  Quality  Style  LotSize
  1  5032     3.00          3.00  0.000000  1989     1.00   7.00    22000
New
Obs   Highway
  1  0.000000
```

Best Subsets Regression: log10Price* versus SqFt, Bedrms, ...

```
Response is log10Price*
                                                  G
                                                  a
                                                  r
                                                  a
                                                  g
                                              B   e     Q   L H
                                            B a   N     u   o i
                                            e t   o     a S t g
                                          S d h   C P Y l t S h
                                          q r r   a o e i y i w
                       Mallows            F m m A r o a t l z a
Vars  R-Sq  R-Sq(adj)      C-p         S  t s s C s l r y e e y
   1  73.4       73.3    416.3  0.095810  X
   1  61.9       61.9    813.5   0.11456                X
   2  80.1       80.0    185.0  0.082917  X             X
   2  79.5       79.4    205.4  0.084135  X           X
   3  82.0       81.9    120.3  0.078893  X           X X
   3  81.6       81.5    133.0  0.079693  X           X     X
   4  83.7       83.6     62.1  0.075068  X           X X   X
   4  83.1       83.0     84.9  0.076572  X           X X X
   5  84.5       84.3     38.3  0.073402  X           X X X X
   5  84.2       84.0     49.2  0.074139  X   X       X X   X
   6  85.1       84.9     20.2  0.072088  X   X       X X X X
   6  84.8       84.6     29.2  0.072711  X         X X X X X
   7  85.3       85.1     13.3  0.071541  X   X     X X X X X
   7  85.3       85.1     15.8  0.071717  X   X   X   X X X X
   8  85.5       85.3      9.5  0.071205  X   X   X X X X X X
   8  85.4       85.2     12.3  0.071402  X   X     X X X X X X
   9  85.6       85.3      8.6  0.071066  X   X   X X X X X X X
   9  85.5       85.3     10.9  0.071231  X   X X X X X X X X
  10  85.6       85.3     10.0  0.071098  X   X X X X X X X X X
  10  85.6       85.3     10.5  0.071132  X X X   X X X X X X X
  11  85.6       85.3     12.0  0.071167  X X X X X X X X X X X
```

Excerpt of Data:

ID	Price	SqFt	Bathrms	GarageNoCars	Pool	Year	Quality	Style	LotSize	Highway	log10Price
72	830	3889	4	3	0	1991	1	7	28378	0	2.91908
101	610	3251	4	3	1	1985	1	1	25570	0	2.78533
102	570	2547	3	3	0	1996	1	1	21789	0	2.75587
103	479	5032	3	3	0	1989	1	7	22000	0	2.68034
104	545	4973	6	3	1	1987	1	7	56139	0	2.7364
105	335	2582	3	2	0	1966	1	2	23256	0	2.52504

Standard Normal Distribution

P(Z < z) (z negative)

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) (z positive)

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

						t_α					
	$\alpha = P(t > t_\alpha)$ = one-tail probability										
ν	**0.100**	**0.050**	**0.025**	**0.010**	**0.009**	**0.008**	**0.007**	**0.006**	**0.005**	**0.004**	**0.003**
1	3.08	6.31	12.71	31.82	35.36	39.78	45.47	53.05	63.66	79.57	106.10
2	1.89	2.92	4.30	6.96	7.35	7.81	8.36	9.05	9.92	11.11	12.85
3	1.64	2.35	3.18	4.54	4.72	4.93	5.18	5.47	5.84	6.32	6.99
4	1.53	2.13	2.78	3.75	3.87	4.01	4.17	4.37	4.60	4.91	5.32
5	1.48	2.02	2.57	3.36	3.46	3.57	3.70	3.85	4.03	4.26	4.57
6	1.44	1.94	2.45	3.14	3.23	3.32	3.43	3.55	3.71	3.90	4.15
7	1.41	1.89	2.36	3.00	3.07	3.16	3.25	3.37	3.50	3.67	3.89
8	1.40	1.86	2.31	2.90	2.97	3.04	3.13	3.23	3.36	3.51	3.70
9	1.38	1.83	2.26	2.82	2.89	2.96	3.04	3.14	3.25	3.39	3.57
10	1.37	1.81	2.23	2.76	2.83	2.89	2.97	3.06	3.17	3.30	3.47
11	1.36	1.80	2.20	2.72	2.78	2.84	2.92	3.00	3.11	3.23	3.39
12	1.36	1.78	2.18	2.68	2.74	2.80	2.87	2.96	3.05	3.17	3.33
13	1.35	1.77	2.16	2.65	2.71	2.77	2.84	2.92	3.01	3.13	3.28
14	1.35	1.76	2.14	2.62	2.68	2.74	2.81	2.88	2.98	3.09	3.23
15	1.34	1.75	2.13	2.60	2.66	2.71	2.78	2.86	2.95	3.06	3.20
16	1.34	1.75	2.12	2.58	2.64	2.69	2.76	2.83	2.92	3.03	3.17
17	1.33	1.74	2.11	2.57	2.62	2.67	2.74	2.81	2.90	3.00	3.14
18	1.33	1.73	2.10	2.55	2.60	2.66	2.72	2.79	2.88	2.98	3.11
19	1.33	1.73	2.09	2.54	2.59	2.64	2.71	2.78	2.86	2.96	3.09
20	1.33	1.72	2.09	2.53	2.58	2.63	2.69	2.76	2.85	2.95	3.07
21	1.32	1.72	2.08	2.52	2.57	2.62	2.68	2.75	2.83	2.93	3.06
22	1.32	1.72	2.07	2.51	2.56	2.61	2.67	2.74	2.82	2.92	3.04
23	1.32	1.71	2.07	2.50	2.55	2.60	2.66	2.73	2.81	2.90	3.03
24	1.32	1.71	2.06	2.49	2.54	2.59	2.65	2.72	2.80	2.89	3.01
25	1.32	1.71	2.06	2.49	2.53	2.58	2.64	2.71	2.79	2.88	3.00
26	1.31	1.71	2.06	2.48	2.53	2.58	2.63	2.70	2.78	2.87	2.99
27	1.31	1.70	2.05	2.47	2.52	2.57	2.63	2.69	2.77	2.86	2.98
28	1.31	1.70	2.05	2.47	2.51	2.56	2.62	2.69	2.76	2.86	2.97
29	1.31	1.70	2.05	2.46	2.51	2.56	2.62	2.68	2.76	2.85	2.96
30	1.31	1.70	2.04	2.46	2.50	2.55	2.61	2.67	2.75	2.84	2.96
31	1.31	1.70	2.04	2.45	2.50	2.55	2.60	2.67	2.74	2.83	2.95
32	1.31	1.69	2.04	2.45	2.49	2.54	2.60	2.66	2.74	2.83	2.94
33	1.31	1.69	2.03	2.44	2.49	2.54	2.60	2.66	2.73	2.82	2.94
34	1.31	1.69	2.03	2.44	2.49	2.54	2.59	2.65	2.73	2.82	2.93
35	1.31	1.69	2.03	2.44	2.48	2.53	2.59	2.65	2.72	2.81	2.93
36	1.31	1.69	2.03	2.43	2.48	2.53	2.58	2.65	2.72	2.81	2.92
37	1.30	1.69	2.03	2.43	2.48	2.52	2.58	2.64	2.72	2.80	2.92
38	1.30	1.69	2.02	2.43	2.47	2.52	2.58	2.64	2.71	2.80	2.91
39	1.30	1.68	2.02	2.43	2.47	2.52	2.57	2.64	2.71	2.80	2.91
40	1.30	1.68	2.02	2.42	2.47	2.52	2.57	2.63	2.70	2.79	2.90
50	1.30	1.68	2.01	2.40	2.45	2.49	2.55	2.61	2.68	2.76	2.87
100	1.29	1.66	1.98	2.36	2.41	2.45	2.50	2.56	2.63	2.71	2.81
200	1.29	1.65	1.97	2.35	2.39	2.43	2.48	2.54	2.60	2.68	2.78
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.46	2.52	2.58	2.66	2.75

Chi-square distribution

$\chi 2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.050	0.010	0.005	0.004	0.003	0.002	0.001	0.0005
1	3.84	6.63	7.88	8.28	8.81	9.55	10.83	12.12
2	5.99	9.21	10.60	11.04	11.62	12.43	13.82	15.20
3	7.81	11.34	12.84	13.32	13.93	14.80	16.27	17.73
4	9.49	13.28	14.86	15.37	16.01	16.92	18.47	20.00
5	11.07	15.09	16.75	17.28	17.96	18.91	20.52	22.11
6	12.59	16.81	18.55	19.10	19.80	20.79	22.46	24.10
7	14.07	18.48	20.28	20.85	21.58	22.60	24.32	26.02
8	15.51	20.09	21.95	22.55	23.30	24.35	26.12	27.87
9	16.92	21.67	23.59	24.20	24.97	26.06	27.88	29.67
10	18.31	23.21	25.19	25.81	26.61	27.72	29.59	31.42

Fisher's F distribution

$F_{\alpha;\nu 1;\nu 2}$

$\alpha = P(F \geq f_{\alpha;\nu 1;\nu 2}) =$ 0.050

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85