



# Université d'Ottawa · University of Ottawa

Faculté des sciences      Faculty of Science  
Mathématiques et de statistique      Mathematics and Statistics

## Sample Midterm MAT 2377: Probability and Statistics for Engineers

Duration: 80 minutes

Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

**You must sign below.**

*Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.*

**By signing below, you acknowledge that you have ensured that you are complying with the above statement.**

\_\_\_\_\_

**This is a closed book examination. One single-sided sheet is permitted. Only a non-programmable and non-graphic calculator is permitted. Record your answer to each question in the table below.**

Question	Answer	Question	Answer	Question	Answer
1		5		9	
2		6		10	
3		7		11	
4		8		12	

**NOTE: At the end of the examination, hand in only this page.**

## Multiple Choice Questions

- [1] 1. Let  $A, B$  and  $C$  be three independent events with  $P(A) = 0.5$ ,  $P(B) = 0.2$  and  $P(C) = 0.3$ . Find the probability that at least one of the  $A, B$  and  $C$  occurs.
- (A) 0.03      (B) 1      (C) 0      (D) 0.72      (E) 0.28.
- [1] 2. Let  $X$  be a random variable with a positive mean such that
- $$\text{Var}[X] = 10 \quad \text{and} \quad E[(X - 5)^2] = 59.$$
- Determine the expected value of  $X$ .
- (A) 5      (B) 12      (C) -10      (D) 10      (E) 30
- [1] 3. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let  $p = 0.7$  be the probability that the student knows the answer and  $q = 0.3$  the probability that the student guesses (when he does not know the answer). Assume that a student who guesses at the answer will be correct with probability 0.2 (there are 5 answers and one of them is correct). What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly ?
- (A) 0.421      (B) 0.427      (C) 0.2      (D) 0.921      (E) 0.6.
- [1] 4. In a sequence of independent tests of integrated circuits, each circuit is rejected with probability 0.2. In a sample of size  $n = 20$ , find the probability that at least 2 integrated circuits are rejected?
- (A) 0.435      (B) 0.931      (C) 0.251      (D) 0.352      (E) 0.558.
- [1] 5. We know that 10% of the components in a particular lot of 50 components are defective. We select 5 components at random from this lot of 50 components. What is the probability that our sample will contain exactly 1 defective component.
- (A) 0.352      (B) 0.029      (C) 0.100      (D) 0.455      (E) 0.055

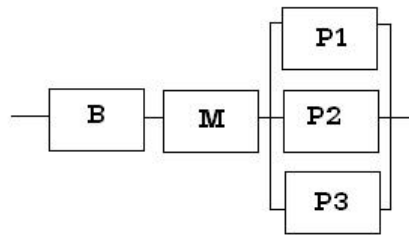
- [1] 6. The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \frac{100}{x^2}, \quad x > 100.$$

Compute the probability that the lifetime will be less than 175 hours.

- (A) 0.5321      (B) 0.0100      (C) 0.6473      (D) 0.1222      (E) 0.4286

- [1] 7. We want to build a computer. Three important components of a computer are the bus, the memory and the processor. Suppose that the reliability (i.e. probability of working) of each of these components are 0.99, 0.95 and 0.6, respectively. To increase the reliability of the processor, we will use an engineering principle of redundancy by assembling three processors in parallel. Assume that components fail independently. What is the reliability of such a computer? An illustration of the system is found below.



- (A) 0.8803      (b) 0.7374      (C) 0.9865      (D) 0.3762      (E) 0.5643

- [1] 8. An air circulation system has 4 filters that need to be cleaned periodically. Let  $X$  be the number of clean filters at any given time. Its cumulative distribution function  $F$  is given below.

$x$	0	1	2	3	4
$F(x)$	0.01	0.10	0.22	0.37	1.00

Find  $P(1 < X \leq 3)$ .

- (A) 0.36      (B) 0.59      (C) 0.27      (D) 0.69      (E) 0.63

- [1] 9. Earthquakes in a given region follow a Poisson process with rate 0.05 per month. What is the probability that we will have at least 2 earthquakes in a period of 12 months?
- A) 0.8781      B) 0.1219      C) 0.3292      D) 0.0988      E) 0.9012

- [1] 10. Consider the following joint probability mass function:

$x$	$y$	$f_{XY}(x, y)$
1	1	$5/20$
1	2	$1/20$
2	1	$2/20$
2	3	$3/20$
3	1	$3/20$
3	2	$1/20$
3	3	$5/20$

We can compute

$$E[X] = 2.15, \quad E[Y] = 1.9, \quad V[X] = 0.7275, \quad V[Y] = 0.89,$$

Determine  $V[3X + 5Y]$ , rounded to the nearest integer.

- A) 40              B) 25              C) 82              D) 7              E) 24
- [1] 11. The probability that a submarine will have a bad propeller is 0.3, the probability that it will have a bad periscope is 0.4, and the probability that it will have both defects is 0.1
- What is the probability that the submarine will have a bad propeller but a good periscope?
- A) 0.1      B) 0.12      C) 0.2      D) 0.4      D) 0.5      E) 0.3

- [1] 12. We program a computer to generate a 4-digit pin at random, allowing repeated digits. The digits are chosen from the set  $\{0,1,2,\dots,9\}$ . Assuming that all possible 4-digit PINs are equally likely of being generated, what is the probability that the computer will generate a 4-digit pin with no repeated digits, i.e. we get 4 distinct digits.

A)  $(6 \times 5 \times 4 \times 3 \times 2)/(10 \times 9 \times 8 \times 7)$

B)  $\binom{6}{4}/\binom{10}{4}$

C)  $\binom{10}{4}/(10 \times 9 \times 8 \times 7)$

D)  $\binom{10}{4}/10^4$

E)  $(10 \times 9 \times 8 \times 7)/10^4$