Do not write in this table.

Question	Q1	Q2	Q3	Q4	Q5	Q6	Total
Maximum points	2 pts	2 pts	2 pts	2 pts	2 pts	2 pts	12 points
Marks obtained	2	2	2	2	2	2	12

1a. Write down the formula for the average value of a function $f(x)$ over the interval $[a, b]$.

$$F_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

1b. Find the average value of the function $f(x) = \dfrac{x^2}{\sqrt{x^3+1}}$ on the interval $[0, 2]$.

$$F_{avg} = \frac{1}{2-0}\int_0^2 \frac{\cancel{x^2}}{\sqrt{u}\cdot 3\cancel{x^2}}\,dx \qquad u^{-1/2} \qquad u^{1/2+\frac{2}{2}}$$

$$= \frac{1}{2}\cdot\frac{1}{3}\int_0^2 \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{6}\int_0^2 2u^{1/2}\,du$$

$$= \frac{1}{6}(x^3+1)^{1/2}\Big|_0^2$$

$$= \frac{1}{3}(8+1)^{1/2} - \frac{1}{3}(0+1)^{1/2}$$

$$= \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

$$\frac{x^2}{\sqrt{x^3+1}}$$

$u = x^3+1$

$du = 3x^2dx$

$\frac{du}{2x^2} = dx$

$$\frac{3}{2} - \frac{1}{2} =$$

$$2(x^3+1)^{1/2}$$

$$\frac{1}{2}(x^3+1)^{-1/2}(2x^2)$$

$$\frac{1^2}{\sqrt{x^3+1}}$$

$u = x^3+1$

$du = 3x^2dx$

$u^{-1/2}$

$$\int \frac{1}{2}\cdot\frac{x^2}{\sqrt{u}\cdot 3x^2}$$

$$\frac{1}{6}\cdot\frac{1}{u^{1/2}}$$

$$\frac{1}{6}\,2u^{1/2}$$

$$= \frac{1}{3}(x^3+1)^{1/2}\Big|^2$$

$$= \frac{1}{3}(9)^{1/2} - \frac{1}{3}(1)^{1/2}$$

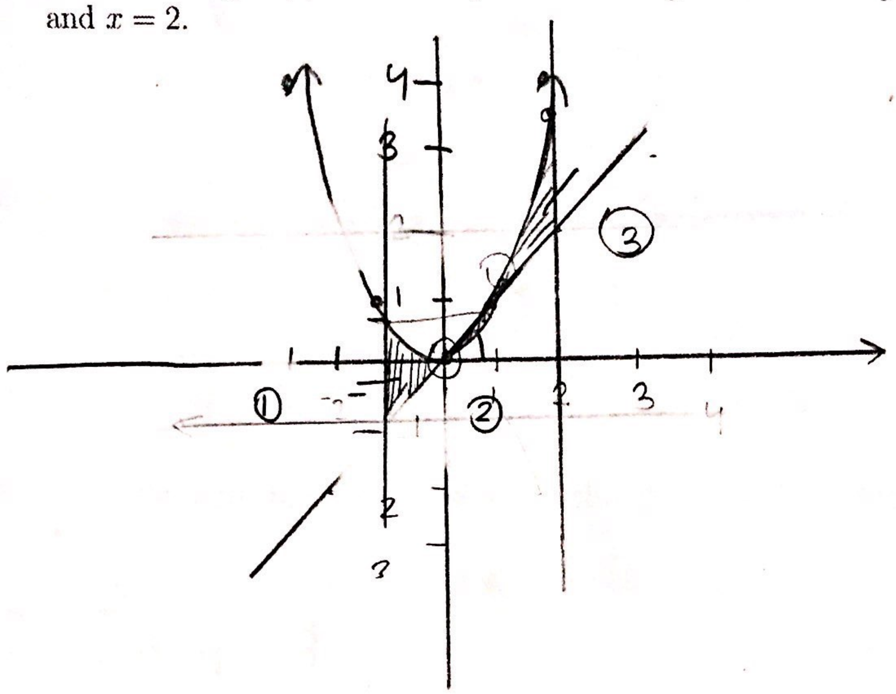
$$= 1 - \frac{1}{3} \qquad -\frac{1}{3}$$

$$= \frac{2}{3}$$

1c. What is its average value? Circle the best response:

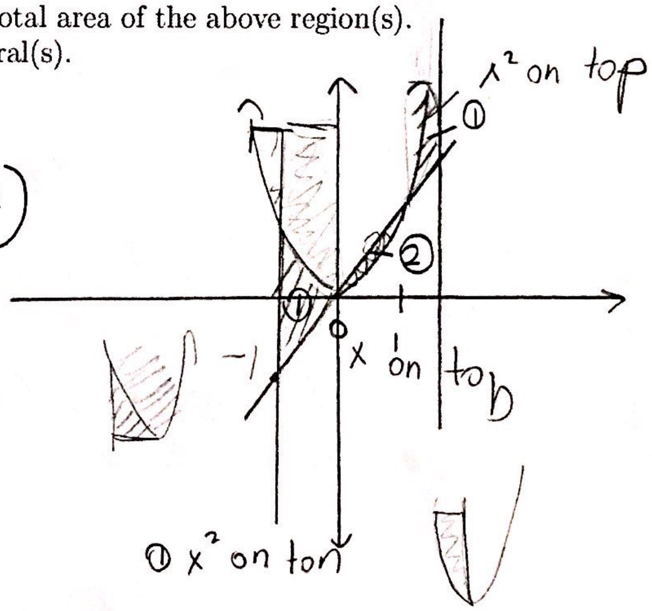
A. $\frac{4}{3}$ **B.** $\frac{2}{3}$ **C.** $\frac{8}{3}$ **D.** $-\frac{10}{3}$ **E.** 2 **F.** 0 **G.** none of the previous answers

2a. Sketch the region(s) bounded by the curves $y = x^2$ and $y = x$ and the lines $x = -1$ and $x = 2$.



2b. Write down the integral(s) needed to find the total area of the above region(s).
Note: You do **not** need to evaluate your integral(s).

$$A = \int_{-1}^{0} (x^2 - x) + \int_{0}^{1} (x - x^2) + \int_{1}^{2} (x^2 - x)$$



P.O.T

$x^2 = x$

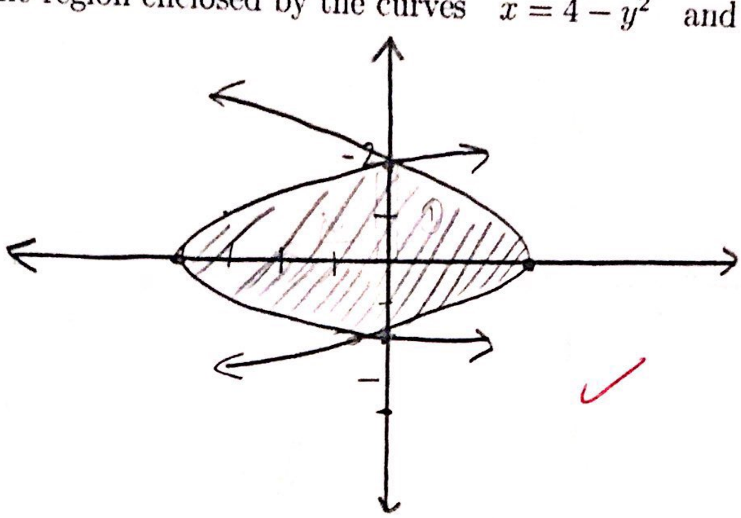
$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad x = +1$

2/2

3a. Sketch the region enclosed by the curves $x = 4 - y^2$ and $x = y^2 - 4$.



$(0) - 4$

$=$

$0 = y^2 - 4$

$0 + 4 = y^2$

$\pm 2 = y$

3b. Write down the integral needed to find the total area of the above region and fully evaluate it.

P.O.T

$4 - y^2 = y^2 - 4$

$8 = 2y^2$

$4 = y^2$

$\pm 2 = y$

$\to$ diff w/ respect to y

$$A = \int_{-2}^{2} (4 - y^2) - (y^2 - 4)$$

$$= \int_{-2}^{2} (4 - 2y^2 + 4)$$

$$= \int_{-2}^{2} (-2y^2 + 8)$$

$$= \left(-\frac{2y^3}{3} + 8x\right)\Big|_{-2}^{2}$$

$$= \left(-\frac{2(8)}{3} + 16\right) - \left(\frac{-2(-8)}{3} - 16\right)$$

$$= -\frac{32}{3} + 32$$

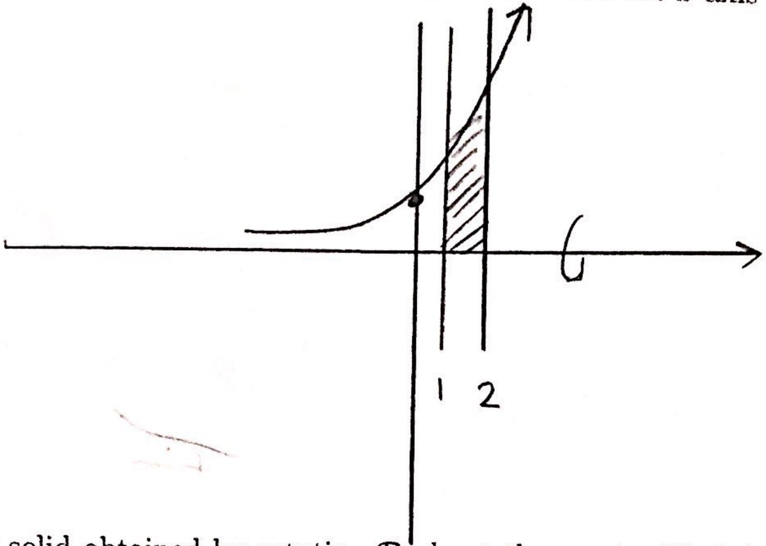
$$= -\frac{32}{3} + \frac{96}{3} = \frac{64}{3}$$

2/2

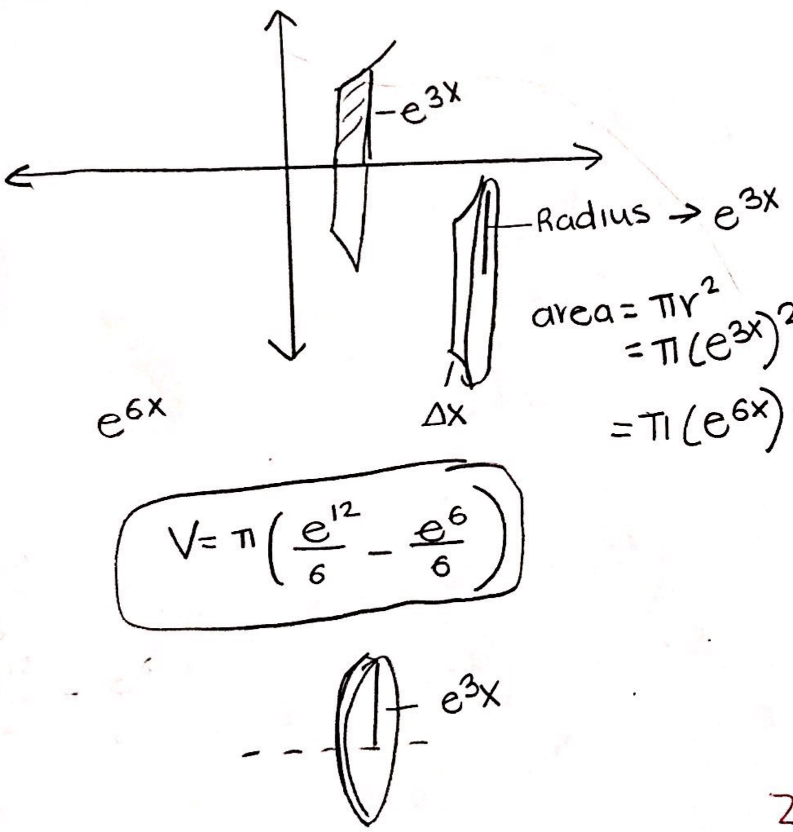
3c. What is the total area? Circle the best response:

A. 21 **B.** $\frac{128}{3}$ **C.** $-\frac{37}{3}$ **D.** 0 **E.** $-\frac{64}{3}$ **F.** $\frac{64}{3}$ **G.** none of the previous answers

4a. Sketch the region $\mathcal{R}$ enclosed by the curve $y = e^{3x}$ and the x-axis and the lines $x = 1$ and $x = 2$.



4b. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ about the x-axis. Find the exact volume of $\mathcal{S}$. Show your work!



area $= \pi r^2$
$= \pi (e^{3x})^2$
$= \pi (e^{6x})$

$V = \int_1^2 A\,dx$

$\frac{e^{6x}}{6} = \frac{e^{6x}(6)}{6}$

$\pi \int_1^2 (e^{6x} - 0)\,dx$

$= \pi \left(\frac{e^{6x}}{6}\right)\Big|_1^2$

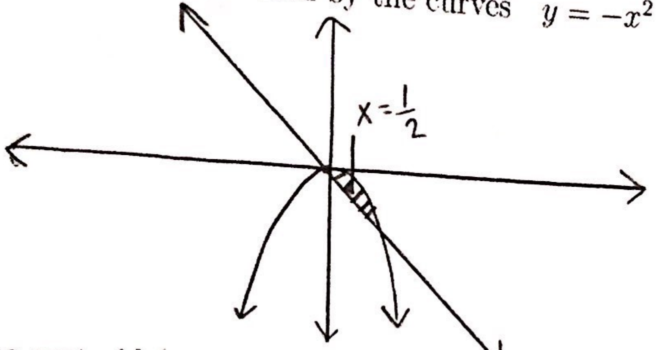
$= \pi \left(\left(\frac{e^{6(2)}}{6}\right) - \frac{e^{6(1)}}{6}\right)$

$= \pi \left(\frac{e^{12}}{6} - \frac{e^{6}}{6}\right)$ //

$$\boxed{V = \pi\left(\frac{e^{12}}{6} - \frac{e^{6}}{6}\right)}$$

2/2

5a. Sketch the region $\mathcal{R}$ bounded by the curves $y = -x^2$ and $y = -x$.



5b. Find the centroid (centre of mass) of $\mathcal{R}$. Show your work!

P.O.I $\quad \frac{1}{A}$

$-x^2 = -x$

$-x^2 + x = 0$

$-x(x-1) =$

$\hookrightarrow x = 0 \quad \hookrightarrow x = 1$

Find Area

$A = \int_0^1 (-x^2 - (-x))$

$= \int_0^1 (-x^2 + x)$

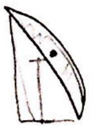
$= \left(-\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1$

$= \left(-\frac{(1)}{3} + \frac{1}{2}\right) - 0$

$= \left(-\frac{2}{6} + \frac{3}{6}\right)$

$= \left(\frac{1}{6}\right) - 0$

$= \frac{1}{6}$ //



$\bar{x} = \frac{1}{A}\int_0^1 x(-x^2 + x)$

$= \frac{1}{\frac{1}{6}}\int_0^1 -x^3 + x^2$

$= 6\left(-\frac{x^4}{4} + \frac{x^3}{3}\right)\Big|_0^1$

$= 6\left(-\frac{1}{4} + \frac{1}{3}\right) - 0$

$= 6\left(-\frac{3}{12} + \frac{4}{12}\right)$

$= 6\left(\frac{1}{12}\right)$

$\bar{x} = \frac{1}{2}$

-1/2

$\bar{y} = \frac{1}{A}\int_0^1 \frac{1}{2}\left((-x^2)^2 - (-x)^2\right)$

$= \frac{1}{\frac{1}{6}}\int_0^1 \frac{1}{2}\left(x^4 - x^2\right)$

$= 3\left(\frac{x^5}{5} - \frac{x^3}{3}\right)\Big|_0^1$

$= 3\left(\frac{1}{5} - \frac{1}{3}\right) - 0$

$= 3\left(\frac{3}{15} - \frac{5}{15}\right)$

$= 3\left(-\frac{2}{15}\right) = -\frac{6}{15} = \frac{-2}{5}$

$\boxed{\bar{x} = \frac{1}{2} \quad \bar{y} = \frac{-2}{5}}$

5c. What is its centroid? Circle the best response:

A. $(-\frac{1}{2}, \frac{2}{5})$ **B.** $(\frac{1}{2}, -\frac{2}{5})$ **C.** $(-\frac{1}{2}, -\frac{2}{5})$ **D.** $(\frac{1}{2}, -\frac{1}{2})$

E. $(-\frac{1}{2}, \frac{1}{2})$ **F.** $(-\frac{1}{2}, -\frac{1}{2})$ **G.** none of the previous answers

6a. Is the integral $I = \int_{-2}^{7} \frac{6}{\sqrt[3]{x+1}}\,dx$ proper or improper? Briefly explain.

→ No, $\sqrt{x+1}=0$ the function does not exist at $x=-1$, there is a discontinuity, improper integral of type 2

$\boxed{x=-1}$

6b. Using appropriate mathematical notation and methods, evaluate the integral:

$$I = \int_{-2}^{7} \frac{6}{\sqrt[3]{x+1}}\,dx$$

Show your work!

$I = \int_{-2}^{-1} \frac{6}{\sqrt[3]{x+1}} + \int_{-1}^{7} \frac{6}{\sqrt[3]{x+1}}$

$u = x+1$
$du = dx$

$= \lim_{t\to -1^-} \int_{-2}^{t} \frac{6}{\sqrt[3]{x+1}}$

$u^{-1/3} + \frac{3}{3}$

$= \lim_{t\to -1^-} \int_{-2}^{t} \frac{6}{u^{1/3}}$

$= \lim_{t\to -1^-} 6 \int_{-2}^{t} \frac{3u^{2/3}}{2}$

$= \lim_{t\to -1^-} 6 \left(\frac{3(x+1)^{2/3}}{2}\right)\Big|_{-2}^{t}$

$= \lim_{t\to -1^-} 6 \left(\cancel{\frac{3(0.000)^{2/3}}{2}}\right) - \left(\frac{3(-1)^{2/3}}{2}\right)$

$= 0 - \frac{3}{2}$

$= -\frac{3}{2}\cdot 6$

$= -9$

$\int_{-1}^{7} \frac{6}{\sqrt[3]{x+1}}$

$= 6\left(\frac{3(x+1)^{2/3}}{2}\right)\Big|_{-1}^{7}$

$= 6\left(\frac{3(8)^{2/3}}{2}\right) - 0$

$= 6\left(\frac{3(4)}{2}\right)$

$= 36$

$\sqrt[3]{64}$ $\sqrt[2]{8^3}$ $\sqrt{64}$

2/2

$I = \int_{-2}^{1} \frac{6}{\sqrt[3]{x+1}} + \int_{-1}^{7} \frac{6}{\sqrt[3]{x+1}}$

$= -9 + 36$

$= 27$

6c. Circle the answer that best describes the integral I.

A. ~~proper,~~ divergent, $I = 27$

B. ~~proper,~~ convergent, $I = 0$

C. improper, convergent, $I = 27$

D. improper, divergent

E. improper, convergent $I = -27$

F. improper, convergent $I = 0$

G. none of the above

(end of Test 1)