MAT 1341 D, Quiz 1

February 4, 2019

Length: 15 minutes.

Professor: Michael Reeks.

Family name:______________________

First name:________________________

Student number:____________________

1	A
2	E
3	B
Total	

PLEASE CAREFULLY READ THESE INSTRUCTIONS:

1. Carefully read each question and **record your responses in the space provided on this page as well as the question page.**
2. You are not allowed to consult your notes or any books. Calculators, phones, and other electronic devices are not allowed.
3. There are three multiple choice questions, each worth 1 point. No partial credit will be awarded. **You must indicate the method you used to select the correct answer; unjustified answers will not be given credit.**

   Record your answers both on the question page and on the title page.

1. Which of the following subsets are subspaces of $M_2(\mathbb{R})$?

A. $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; a + d = 0 \right\}$

B. $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; ad = 1 \right\}$

C. $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; a,\, b,\, c,\, d \text{ are integers} \right\}$

D. $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; ad - bc = 0 \right\}$

E. $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; a = 1 \right\}$

F. None of these subsets are subspaces.

Another way to write the set A is

$$\left\{ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \right\}.$$

This is the span of the matrices

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

and so is a subspace (any spanning set is a subspace).

Set B is not closed under scalar multiplication: multiplying any matrix in this set by $c \in \mathbb{R}$ *will also multiply both sides of the equation* $ad = 1$ *by* c*. It also does not contain the zero vector.*

Set C is not closed under scalar multiplication: multiply any matrix in this set by any non-whole number.

Set D is not closed under addition. Consider $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ *and* $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

Set E does not contain the zero vector, and is closed under neither addition nor scalar multiplication.

2. Let $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 0\}$. Which of the following subsets of $\mathbb{R}^3$ are a spanning set for W?

A. $\{(0, 0, 0)\}$

B. $\{(1, 1, -1)\}$

C. $\{(-1, 1, 0), (1, -1, 0)\}$

D. $\{(1, 0, 1)\}$

E. $\{(-1, 1, 0), (1, 0, 1)\}$

F. $\{(1, 1, -1), (-1, 1, 0)\}$

We have $x = -y + z$, *so we can also write this set as* $\{(-y + z, y, z) | y, z \in \mathbb{R}\}$. *Thus* $W = \{y(-1, 1, 0) + z(1, 0, 1) | y, z \in \mathbb{R}\} = \text{span}\{(-1, 1, 0), (1, 0, 1)\}$.

3. Let $\mathbb{P}_2 = \{\, p \mid p(x) = a + bx + cx^2, \text{ where } a, b, c \in \mathbb{R}\}$, the vector space consisting of polynomials of degree less than or equal to 2 with real coefficients.

Consider the following subset of $\mathbb{P}_2$:

$$S = \{x^2 - 1,\ x^2 + 1,\ x - 1,\ x + 1\}$$

Which of the following statements about S is true?

I. S is linearly dependent
II. S is linearly independent
III. S spans $\mathbb{P}_2$
IV. S is a basis of $\mathbb{P}_2$

A. (I) and (II)
B. (I) and (III)
C. (II) and (IV)
D. (II) and (III)
E. (I), (III) and (IV)
F. (III) and (IV)

To test whether or not S is linearly dependent, we look for a solution to

$$a_1(x^2 - 1) + a_2(x^2 + 1) + a_3(x - 1) + a_4(x + 1) = 0 \quad \textit{for all } x.$$

Multiplying this out and collecting terms with the same degree, we have

$$(a_1 + a_2)x^2 + (a_3 + a_4)x + (a_2 + a_4 - a_1 - a_3) = 0.$$

In order for this to be true for all x, we must have that each of the expressions in parentheses is equal to 0. Hence

$$a_1 + a_2 = 0$$
$$a_3 + a_4 = 0$$
$$a_2 + a_4 - a_1 - a_3 = 0.$$

Solving the first two equations gives $a_1 = -a_2$ and $a_3 = -a_4$. Plugging this into the last equation gives

$$a_2 + a_4 + a_2 + a_4 = 0 \Rightarrow a_2 = -a_4.$$

Here, we had a system of three equations in four unknowns, so we got infinitely many solutions instead of just one. All we have to do is choose a value for a_4, and the rest is forced: for $a_4 = 1$, we have $a_2 = -1$, $a_3 = -1$, $a_1 = 1$, and we can check that

$$(x^2 - 1) - (x^2 + 1) - (x - 1) + (x + 1) = 0.$$

Hence S is LD. This automatically means that (II) and (IV) are false, so the answer is B.