CLASS NUMBER:

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1322 Winter 2019
Dr. A. Czajkowski

LAST NAME: ______________________

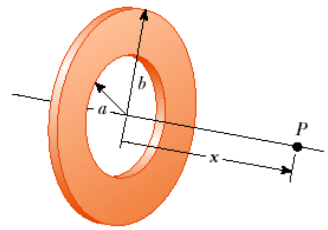
Assignment 3: ELECTRICITY

Released: Friday Feb.1 Due: Friday Feb.8 6:00PM

1 Calculate the electric potential at point P on the axis of the annulus shown on the right which has a uniform charge density σ.

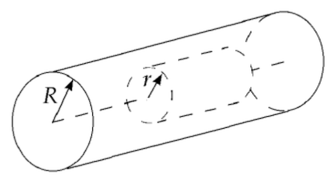
$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} \quad \text{where} \quad dq = \sigma dA = \sigma 2\pi r dr$$

$$V = 2\pi\sigma k_e \int_a^b \frac{r dr}{\sqrt{r^2 + x^2}} = \boxed{2\pi k_e \sigma\left[\sqrt{x^2 + b^2} - \sqrt{x^2 + a^2}\right]}$$



2 Obtain the expression for the electric potential at distance r from the centre of the charged cylindrical conductor of radius R and infinite length.

If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length *L* and radius *r*, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\mathbf{E} \cdot d\mathbf{A} = EdA\cos 90.0° = 0$. The curved surface has $\mathbf{E} \cdot d\mathbf{A} = EdA\cos 0°$, and *E* must be the same strength everywhere over the curved surface.



Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\text{Curved Surface}} dA = \frac{\rho \pi r^2 L}{\epsilon_0}$. Now the lateral surface area of the cylinder is $2\pi r L$: $E(2\pi r)L = \frac{\rho \pi r^2 L}{\epsilon_0}$. Thus, $\mathbf{E} = \boxed{\frac{\rho r}{2\epsilon_0} \text{ radially away from the cylinder axis}}$.

Potential is given by $\int_0^r \frac{\rho}{2\varepsilon} r dr = \frac{\rho}{4\varepsilon} r^2$

Outside of the cylinder V=$\int_R^r \frac{\rho R^2}{2\varepsilon}\frac{1}{r} dr = \frac{\rho R^2}{2\varepsilon} ln\frac{r}{R}$

3 Two conducting spheres of different radii R1and R2 are connected via thin wire made of perfect conductor, and charged . Calculate the ratio of their charges, and the ratio of their electric field magnitudes as functions of R1 and R2.

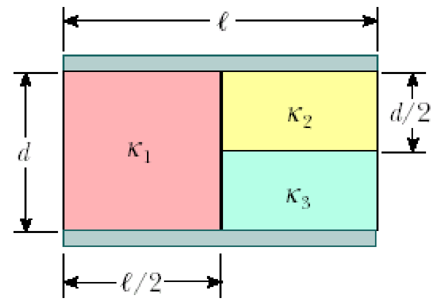
THIS PROBLEM WAS SOLVED IN DETAIL IN CLASS

$$\frac{Q_1}{Q_2} = \frac{E_1}{E_2} = \frac{r_2}{r_1}$$

4 Two capacitors, $C1 = 5.00\ \mu F$ and $C2 = 12.0\ \mu F$, are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the equivalent capacitance of the combination? What are (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?

a) C=C1+C2= 17μF *b)* $\mathcal{E} = \Delta V1 = \Delta V2 = 9V$ *c) Q1=45µC* *Q2=108µC*

5 A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure. You may assume that $\ell >> d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00\ cm^2$, $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.



(a) $$C_1 = \frac{\kappa_1 \epsilon_0 A/2}{d};\ C_2 = \frac{\kappa_2 \epsilon_0 A/2}{d/2};\ C_3 = \frac{\kappa_3 \epsilon_0 A/2}{d/2}$$

$$\left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

$$C = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \boxed{\frac{\epsilon_0 A}{d}\left(\frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)}$$

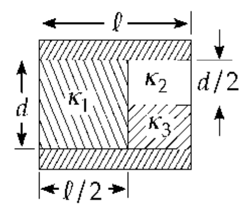
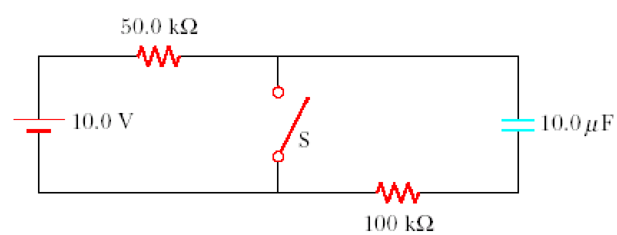


FIG. P26.61

(b) Using the given values we find: $C_{total} = 1.76 \times 10^{-12}\ F = \boxed{1.76\ pF}$.

6 In the circuit below, the switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.



(a) $\tau = RC = (1.50 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ F) = \boxed{1.50\ s}$

(b) $\tau = (1.00 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ F) = \boxed{1.00\ s}$

(c)The battery carries current $\frac{10.0\ V}{50.0 \times 10^3\ \Omega} = 200\ \mu A$.

The $100\ k\Omega$ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\frac{10.0\ V}{100 \times 10^3\ \Omega}\right) e^{-t/1.00\ s}$.

So the switch carries downward current $\boxed{200\ \mu A + (100\ \mu A) e^{-t/1.00\ s}}$.