Econ 301: Assignment 2–Solutions

Graphs are attached at the end.

1. Let x be Niki's optimal consumption bundle under the original price, and let y be Niki's optimal consumption bundle under the new price. Then

$$x_1 = \frac{m}{2p_1} = 25$$

$$x_2 = \frac{m}{2p_2} = 25$$

$$y_1 = \frac{m}{2p_1'} = 12.5$$

$$y_2 = \frac{m}{2p_2} = 25$$

In order to calculate CV, we need to calculate m' such that the optimal consumption bundle under (p_1', p_2, m') gives Niki the same utility as x. Denote this consumption bundle by z. Then

$$z_1 = \frac{m'}{2p_1'} = \frac{m'}{8}$$

$$z_2 = \frac{m'}{2p_2} = \frac{m'}{4}$$

$$(\frac{m'}{8})^{1/2}(\frac{m'}{4})^{1/2} = 25^{1/2}25^{1/2} = 25$$

$$m' = 25\sqrt{32} \approx 141.42$$

and

$$CV = m' - m \approx 141.42 - 100 = 41.42.$$

In order to calculate EV, we need to calculate m'' such that the optimal consumption bundle under (p_1, p_2, m'') gives Niki the same utility as y. Denote this consumption bundle by z'. Then

$$z_1' = \frac{m''}{2p_1} = \frac{m''}{4}$$

$$z_2' = \frac{m''}{2p_2} = \frac{m''}{4}$$

$$(\frac{m''}{4})^{1/2}(\frac{m''}{4})^{1/2} = 12.5^{1/2}25^{1/2} \approx 17.68$$

$$m'' \approx 17.68(4) = 70.72$$

and

$$EV = m - m'' \approx 100 - 70.72 = 29.28.$$

2. a. The price elasticity is

$$\epsilon = \frac{dx}{dp} \cdot \frac{p}{x} = \frac{-2m}{p^3} \cdot \frac{p}{m/p^2} = -2.$$

Since $|\epsilon| = 2 > 1$, demand is elastic. (Note that the price elasticity is constant for this demand function.)

b. Given that demand is elastic ($|\epsilon| > 1$), the revenue from this product increases when the price drops.

c. The income elasticity is

$$\eta = \frac{dx}{dm} \cdot \frac{m}{x} = \frac{1}{p^2} \cdot \frac{m}{m/p^2} = 1.$$

Thus, $\eta > 0$, which shows that this is a normal good. However, this good is neither a luxury good (for which $\eta > 1$) nor a necessary good (for which $\eta < 1$). An income elasticity of 1 indicates that preferences are homothetic, which is the special case between a luxury and a necessary good.

3. a. Setting demand equal to supply, and given that $P_D = P_S = P$, we have $120 - P = 2P$, or $P^* = 40$. The equilibrium quantity is $Q^* = D(P^*) = 120 - 40 = 80$.

b. The supply curve shifts up by 3. The equilibrium price can be found from $D(P_D) = S(P_S)$ and
$P_D = P_S + t$, or

$$120 - (P_S + 3) = 2P_S,$$

which gives $P'_S = 39$. Therefore, $P'_D = P'_S + t = 39 + 3 = 42$, and the equilibrium quantity is $Q' = D(P'_D) = 120 - 42 = 78$. Since $P_{D'} - P^* = 42 - 40 = 2$, \$2 per unit of the tax is passed on to the consumer.

c. The demand curve shifts down by 3. The equilibrium price can be found from $D(P_D) = S(P_S)$ and $P_D - t = P_S$, or

$$120 - P_D = 2(P_D - 3),$$

which gives $P'_D = 42$. Therefore, $P'_S = P'_D - t = 42 - 3 = 39$, and the equilibrium quantity is $Q' = D(P'_D) = 120 - 42 = 78$. The answers in parts b. and c. are exactly the same: the equilibrium outcome does not depend on whether the supplier or the demander pays the tax.

d. In order to calculate these quantities, observe that the vertical intercept of the demand curve is $\bar{P} = 120$ and the vertical intercept of the supply curve is $\underline{P} = 0$. The initial consumer's surplus is

$$CS = \frac{1}{2}(\bar{P} - P^*)Q^* = \frac{1}{2}(120 - 40)80 = 3200.$$

After the tax is imposed, the consumer's surplus becomes

$$CS' = \frac{1}{2}(\bar{P} - P'_D)Q' = \frac{1}{2}(120 - 42)78 = 3042,$$

which makes the loss in the consumer's surplus

$$3200 - 3042 = 158.$$

The initial producer's surplus is

$$PS = \frac{1}{2}(P^* - \underline{P})Q^* = \frac{1}{2}(40 - 0)80 = 1600.$$

After the tax is imposed, the producer's surplus becomes

$$PS' = \frac{1}{2}(P'_S - \underline{P})Q' = \frac{1}{2}(39 - 0)78 = 1521,$$

which makes the loss in the producer's surplus

$$1600 - 1521 = 79.$$

The collected tax revenue is

$$T = tQ' = 3(78) = 234.$$

The deadweight loss is the total loss in the consumer's surplus and the producer's surplus minus the tax revenue. Therefore,

$$DWL = 158 + 79 - 234 = 3.$$

4. a. Use the tangency condition and the production function to calculate the conditional input demands x_1 and x_2, in order to calculate the cost function (which is based on cost minimization).

<u>Tangency condition</u>:

$$\frac{MP_1}{MP_2} = \frac{w_1}{w_2}$$

which gives

$$\frac{x_2}{x_1} = \frac{w_1}{w_2}$$

<u>Production function</u>:

$$x_1 x_2 = y$$

Solve the tangency condition and the production function simultaneously for input demands x_1 and x_2. From the production function, $x_1 = \frac{y}{x_2}$. Using the tangency condition this gives

$$\frac{x_2}{\frac{y}{x_2}} = \frac{w_1}{w_2}.$$

Thus, $\frac{x_2^2}{y} = \frac{w_1}{w_2}$ or, equivalently, $x_2^2 = y\frac{w_1}{w_2}$. This yields the conditional input demand for input 2

$$x_2 = y^{1/2}(\frac{w_1}{w_2})^{1/2}.$$

Now we can calculate $x_1 = \frac{y}{x_2}$ as

$$x_1 = y^{1/2}(\frac{w_2}{w_1})^{1/2},$$

which is the conditional input demand for input 1.

The cost function is given by $C(y, w_1, w_2) = w_1x_1 + w_2x_2$, and thus using the conditional input demands we get

$$C(y, w_1, w_2) = 2\sqrt{yw_1w_2}.$$

b. Profit maximization requires to find quantity y that maximizes the profit function $\Pi(y)$, where y is the firm's supply. Given $\Pi(y) = py - C(y, w_1, w_2) = py - 2\sqrt{yw_1w_2}$, use the first-order condition

$$\frac{d\Pi(y)}{dy} = 0.$$

This gives

$$p = \frac{\sqrt{w_1w_2}}{\sqrt{y}}.$$

Now we can solve for y to get the supply function. We get

$$\sqrt{y} = \frac{\sqrt{w_1w_2}}{p},$$

and the firm's supply function is

$$y = \frac{w_1w_2}{p^2}.$$

Graphs for question 3: