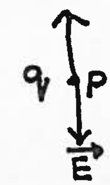
Solutions
Assignment 2
Physics 205/2

Q1

What must be the charge on a particle of mass 2 gram for it to remain stationary in the laboratory when placed in a downward directed electric field of 500 N/C? (ans: -3.92×10^{-5} C)

Solution: Let q be the charge on the particle P

Electric Field $\rightarrow \vec{E} = 500 \frac{N}{C}$ (downward)



Downward gravitational force on P

$$= (0.002)g = 0.002 \times 9.8$$
$$= 0.0196 \, N$$

For the particle to be stationary under the gravitational force, the electrical force on the particle must be equal and opposite to the gravitational force (weight). Since the electrical field is acting downward, the charge on the particle must be negative to yield electrical force upward. The magnitude of the charge is given by,

$$-qE = mg$$

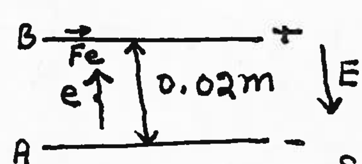
$$\text{or } -q = \frac{mg}{E} = \frac{0.0196}{500} = 3.92 \times 10^{-5} C$$

$$\therefore q = -3.92 \times 10^{-5} C$$

Q2

A uniform electric field exists in a region between two oppositely charged parallel plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2 cm distant from the first, in a time interval of 1.5×10^{-8} s. (a) Find the electric field, (b) find the velocity of the electron when it strikes the second plate. (ans: 1.01×10^{3} N/C)

Solution: Time taken by electron to travel from plate A to B $= 1.5 \times 10^{-8}$ s



The electron travels a distance of 0.02 m in time 1.5×10^{-8} s in a straight line. Therefore

$$S = v_0 t + \frac{1}{2} a t^2 \text{ gives}$$

$$0.02 = 0 + \frac{1}{2} a (1.5 \times 10^{-8})^2 \quad \text{--- (1)}$$

From eq. (1) we get

$$a = \frac{2 \times (0.02)}{(1.5 \times 10^{-8})^2} = 1.78 \times 10^{14}\ m/s^2$$

Electrical force F_e acting on the electron is

$$F_e = eE = m_e a \qquad m_e \rightarrow \text{mass of the electron}$$

$$\therefore (1.6 \times 10^{-19}) E = (9.11 \times 10^{-31})(1.78 \times 10^{14})$$

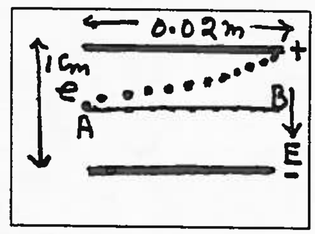
$$\therefore E = \frac{(9.11 \times 10^{-31})(1.78 \times 10^{14})}{(1.6 \times 10^{-19})} = 1.01 \times 10^{3} \frac{N}{C}$$

(b) To find the speed of electron when it reaches the plate B is given by

$$v = v_0 + at$$

$$v = 0 + (1.78 \times 10^{14})(1.5 \times 10^{-8}) = 2.67 \times 10^{6}\ m/s$$

Q3



An electron is projected with an initial velocity $v_0 = 10^7$ m/s into the uniform field between the parallel plates as shown in the figure. The direction of the field is vertically downward and the field is zero except in space between the plates. The electron enters the field at a point midway between the plates. If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electrical field. (ans: 1.43×10^4 N/C).

<u>Solution</u>: Force on the electron, $F_e = eE$ (acting upward)

$\therefore$ Acceleration of the electron upward towards the upper plate,

$$a = \frac{eE}{m} \quad (1)$$

Time taken by electron to travel from point A to B is,

$$t = \frac{d}{v_0} = \frac{0.02}{10^7} = 2 \times 10^{-9}\ s.$$

During this time the electron is deflected by .5 cm due to acceleration caused by the electrical force

The acceleration is given by

$$S = v_0 t + \frac{1}{2} a t^2 \quad (\text{vertical direction})$$

$$0.005 = 0 + \frac{1}{2} a (2 \times 10^{-9})^2 \quad (2)$$

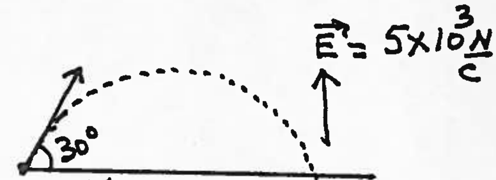
From eq. (2), $a = \frac{2 \times (0.005)}{(2\times10^{-9})^2} = 2.5 \times 10^{15}\, m/s^2$

Therefore from eq. (1), the electric field is given by

$$E = \frac{ma}{e} = \frac{(9.11\times10^{-31})(2.5\times10^{15})}{(1.6\times10^{-19})} = 1.423\times^{4}\, N/C$$

Q.4

An electron is projected into a uniform electric field of 5000 N/C. The direction of the field is vertically upward. The initial velocity of the electron is 10^7 m/s, at an angle of 30° above the horizontal, (a) find the maximum distance the electron rises vertically above the horizontal, (b) after what horizontal distance does the electron return to its original elevation? (c) Draw a sketch the trajectory of the electron. (ans: 1.4 cm, 9.87 cm)



Solution: Since the field is acting upward, the force F_e on the electron will be vertically downward,

$$F_e = eE = (1.6\times10^{-19})(5\times10^{3}) = 8\times10^{-16} N$$

(a) downward acceleration of electron, $a = \frac{F_e}{m} = \frac{8\times10^{-16}}{9.11\times10^{-31}} = 0.88\times10^{15}\, \frac{m}{s^2}$

vertical Component of v_0 is $v_{0y} = v_0 \sin 30^\circ = 10^7 \times \frac{1}{2} = 0.5\times10^{7} m/s$

Maximum vertical distance is given by,

$$v_y^2 - v_{0y}^2 = 2ay$$

$$0 - v_{0y}^2 = -2a\, y_{max} \quad \text{or} \quad y_{max} = \frac{(v_{0y})^2}{2a} = \frac{(0.05\times10^{7})^2}{2(0.88\times10^{15})} = 1.4\times10^{-2} m = 1.4 Cm.$$

(b) Time taken by electron to obtain maximum height is given by, $v = v_0 + at$

$$0 = (0.5\times10^{7}) - (0.88\times10^{15})t$$

$$\text{or } t = \frac{0.5\times10^{7}}{(0.88\times10^{15})} = 5.7\times10^{-9} s.$$

Electron will take the same time to come down,

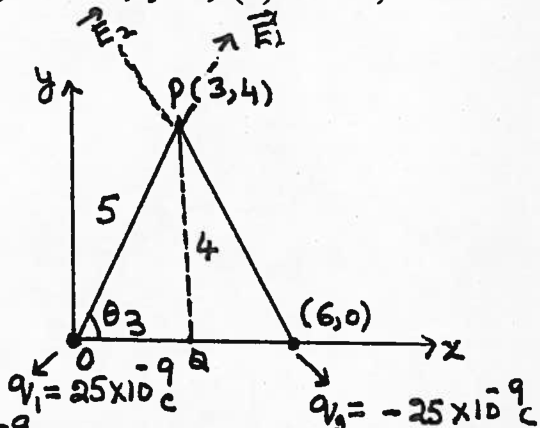
$\therefore$ Total time taken $= 2\times5.7\times10^{-9} = 1.14\times10^{-8}$ s.

Horizontal distance traveled by electron during this time is

$$x = (2t)(v_0 \cos 30^\circ) = (1.14\times10^{-8})(10^{7})(0.866) = 9.87 \text{ Cm}$$

(c) The path of the electron will be a parabola.

Q5 In a rectangular coordinate system a charge of $25\text{x}10^{-9}$ C is placed at the origin of coordinates, and a charge of $-25\text{x}!0^{-9}$ is placed at the point x=6m, y=0. What is the electric field at (a) x=3 m, y=0, (b) x=3m, y=4m? (ans: 50 N/C, 10.8 N/C along x-axis)



Solution: Electrical field at the

(a) point Q(x=3, y=0) is,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

where $\vec{E}_1 \rightarrow$ Field due to q_1

and $\vec{E}_2 \rightarrow$ Field due to q_2

$$\vec{E}_1 = E_{1x}\hat{i} + E_{1y}\hat{j} = \frac{(9\times10^{9})(25\times10^{-9})}{3^2}\hat{i} + 0\hat{j}$$

$$\vec{E}_2 = E_{2x}\hat{i} + E_{2y}\hat{j} = \frac{(9\times10^{9})(25\times10^{-9})}{3^2}\hat{i} + 0\hat{j}$$

$$\therefore \vec{E}(Q) = \vec{E}_1 + \vec{E}_2 = \frac{2(9\times10^{9})(25\times10^{-9})}{3^2}\hat{i} = 50\text{N/c along x-axis.}$$

(b) At the point P (x = 3m, y = 4m)

$$\vec{E}(P) = \vec{E}_1(P) + \vec{E}_2(P)$$

$$\vec{E}_1 = E_{1x}\hat{i} + E_{1y}\hat{j} = \frac{(9\times10^{9})(25\times10^{-9})}{(5)^2}\cos\theta\,\hat{i} + \frac{(9\times10^{9})(25\times10^{-9})}{(5)^2}\sin\theta\,\hat{j}$$

$$\cos\theta = 3/5,\ \sin\theta = \frac{4}{5}$$

$$\therefore \vec{E}_1 = \frac{(9\times10^{9})(25\times10^{-9})}{25}\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$$

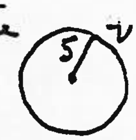
Similarly, $$\vec{E}_2 = \frac{(9\times10^{9})(25\times10^{-9})}{25}\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$$

$$\therefore \vec{E}(P) = \vec{E}_1(P) + \vec{E}_2(P) = 9\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} + \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$$

$$= \frac{9\times6}{5}\hat{i} = 10.8\ \text{N/c in the x-direction}$$

Q.6 How many excess electrons must be added to an isolated spherical conductor 10 cm in diameter to produce a field just outside the surface whose velocity is 1300 N/C. (ans: $2.25\text{x}10^{9}$ electrons)

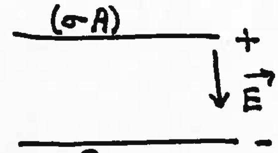
Solution: If q is the charge on the sphere, the field just outside the sphere, $E = \frac{kq}{r^2}$



$$\text{or}\quad 1300 = \frac{(9\times10^{9})q}{(.05)^2} \quad\text{or}\quad q = \frac{1300(0.05)^2}{9\times10^{9}} = 3.6\times10^{-10}\text{c}$$

No. of excess electrons $= 3.6\times10^{-10}/1.6\times10^{-19} = 2.25\times10^{9}$ electrons

Q.7 The electric field in the region between a pair of oppositely charged plane parallel plates, each 100 cm^2 in area, is 10^4 N/C. What is the charge on each plate? (ans: 8.85×10^{-10} C)



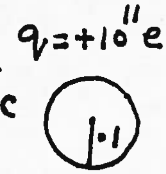
Solution: Electric field between the plates, $E = 10^4 = \frac{\sigma}{\epsilon_0}$

$\therefore \sigma = 10^4 \times \epsilon_0 = 10^4 \times (8.85 \times 10^{-12}) = 8.85 \times 10^{-8}$ C/m^2

Area of each plate $= 100\ Cm^2 = 10^{-2}\ m^2$

$\therefore$ Charge on the plate $= \sigma A = (8.85 \times 10^{-8}) \times 10^{-2} = 8.85 \times 10^{-10}$ C

Q8 An isolated metal sphere of 10 cm radius is deprived of 10^{11} electrons. Find the electric intensity at the surface and at I m from the center. (ans: 14400 N/C, 144N/C)



Solution: Charge on the sphere

$q = 10^{11} \times e = 10^{11} \times (1.6 \times 10^{-19}) = 1.6 \times 10^{-8}$ C

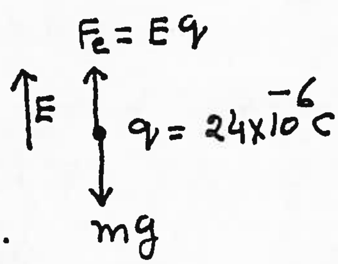
Electric intensity at the surface,

$$E = \frac{(9 \times 10^9)(1.6 \times 10^{-8})}{(0.1)^2} = 1.44 \times 10^4 \text{ N/C}$$

Electric Intensity at a distance of 1 m from the center is,

$$E = \frac{(9 \times 10^9)(1.6 \times 10^{-8})}{(1)^2} = 1.44 \times 10^2 \text{ N/C}.$$

Q9 An object having a net charge of 24 μC is placed in a uniform electric field of 610 N/C directed vertically. What is the mass of this object if it floats in the field? (ans: 1.5 gram)



Solution: If the object floats in the field, the electrical force on the object must be equal and opposite to the weight. Since the charge is +ve the field should act vertically upward.

$Eq = mg$

or $(610)(24 \times 10^{-6}) = m(9.8)$

$$\therefore m = \frac{610 \times 24 \times 10^{-6}}{9.8} = 1.5 \times 10^{-3} \text{ kg}$$

$= 1.5$ gram

Q10

An electron and a proton are each placed at rest in an electric field of 520 N/C. Calculate the speed of each particle 48 ns after being released. ($v_e = 4.4\times10^{6}$ m/s, $v_p = 2.4\times10^{3}$ m/s)

Solution: Electron has negative charge and moves opposite to the direction of electric field

Force on electron $= Ee = 520\times(1.6\times10^{-19})$
$= 8.32\times10^{-17}$ N

$\therefore$ Acceleration of electron $\rightarrow a_e = \frac{F_e}{m_e} = \frac{8.32\times10^{-17}}{9.11\times10^{-31}}$
$= 9.13\times10^{13}$ m/s^2

Speed of electron after 48 ns $\rightarrow v_e = a_e t$
$= 9.13\times10^{13}\times(48\times10^{-9})$
$= 4.4\times10^{6}$ m/s

For proton — $m_p = 1.67\times10^{-27}$ kg, Charge $q_p = 1.6\times10^{-19}$ C

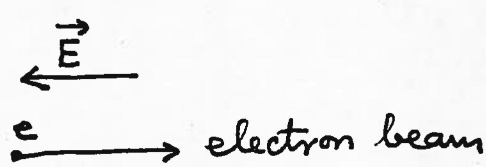
$\therefore a_e = \frac{F_p}{m_p} = \frac{(520)(1.6\times10^{-19})}{1.67\times10^{-27}} = 5.0\times10^{10}$ m/s^2

$\therefore$ speed of proton after 48 ns $= 5.0\times10^{10}\times(48\times10^{-9})$
$= 2.4\times10^{3}$ m/s

Q11

The electrons in a particle beam each have a kinetic energy of 1.60×10^{-17} J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10 cm? (ans: 1.00×10^{3} N/C in the direction of the beam)

Solution: The electric field $\vec{E}$ acts in a direction opposite to the direction of the beam to stop the electrons.



Kinetic energy of electrons $\rightarrow T_e = \frac{1}{2} m_e v_e^2 = 1.6\times10^{-17}$ J.

Force on electron opposing the motion $= E\times(1.6\times10^{-19})$ N.

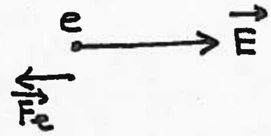
$\therefore$ K.E of electron $\rightarrow T_e =$ work done $= E(1.6\times10^{-19})(0.1)$

or $1.6\times10^{-17} = 1.6\times10^{-20} E \quad \therefore \quad E = 1000$ N/C (in beam direction)

Q12

An electron moves at $3\text{x}10^6$ m/s into a uniform electric field of magnitude 1000 N/C. The field is parallel to the electron's velocity and acts to decelerate the electron. How far does the electron travel before it is brought to rest? (2.56×10^{-2} m)

<u>Solution:</u> Electrical force on electron will be in a direction opposite to the field.



$$\vec{F_e} = e\vec{E} = (-1.6\times10^{-19})(10^3)$$
$$= -1.6\times10^{-16} N \text{ (opposite to the direction of } \vec{E})$$

The electron will therefore decelerate.

$$\therefore \text{ acceleration } a = -\frac{1.6\times10^{-16}}{9.11\times10^{-31}} = 1.76\times10^{14}\ m/s^2$$

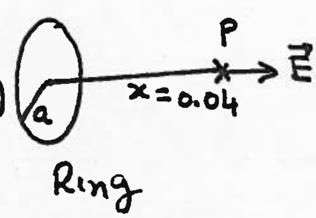
$\therefore$ Distance traveled before electron comes to rest.

$$v^2 - v_0^2 = 2ax$$
$$0 - (3.0\times10^6)^2 = 2(-1.76\times10^{14}).\,x$$
$$\therefore \text{ Distance} \rightarrow x = \frac{(3\times10^6)^2}{2(1.76\times10^{14})} = 2.56\times10^{-2}\ m$$

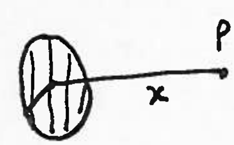
Q13

. A uniformly charged ring and a uniformly charged disc each have a charge of +25 μC and a radius of 3.0 cm. For each of these charged objects, determine the electric field at a point along the axis 4.0 cm from the center of the object. ($7.2\times10^7 \frac{N}{C}$, 10^7 N/C)

<u>Solution:</u> <u>Ring</u>: Electric field along the axis of the ring is given by



$$E_x = \frac{k\,x\,Q}{(x^2+a^2)^{3/2}} = \frac{(9.0\times10^9)(0.0\ )(25\times10^{-6})}{[(0.04)^2+(0.03)^2]^{3/2}}$$
$$= \frac{9.0\times10^3}{(0.05)^3} = 7.2\times10^7\ N/C$$

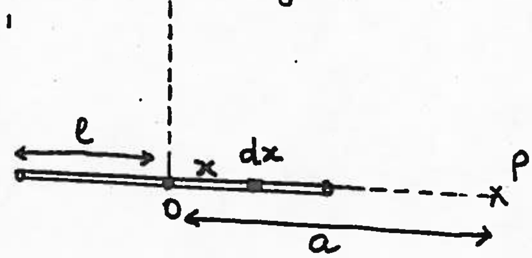


For <u>DISC</u>

$$E_x = 2\pi k\sigma\left(\frac{x}{|x|} - \frac{x}{(x^2+a^2)^{1/2}}\right)$$
$$= 2\pi(9.0\times10^9)\left(\frac{25\times10^{-6}}{\pi(0.03)^2}\right)\left[1 - \frac{0.04}{[(.04)^2+(.03)^2]^{1/2}}\right]$$
$$= 5\times10^8\left[1-\frac{4}{5}\right] = 10^8\ N/C$$

Q.14 A rod 40 cm long is uniformly charged and has a total energy of –22 μC. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36 cm from its center.. (2.21×10^{6} N/C opposite to direction of axis)

<u>Solution</u>: Let l be the length of the rod and a be the distance of P from the Center of the rod.



If dx is an element of the rod at a distance x from the Center of the rod and λ be the charge per unit length of the rod,

Charge on the element $dx = \lambda dx$

Field at P due to the charge $\lambda dx = \dfrac{k\lambda dx}{(a-x)^2}$ (along OP)

$\therefore$ Field at P due to whole rod $= \displaystyle\int_{-l/2}^{+l/2} \frac{k\lambda dx}{(a-x)^2}$ (along OP) (1)

$$= k\lambda\left[-\frac{1}{(a-x)}\right]_{-l/2}^{+l/2}$$

$$= k\lambda\left[-\frac{1}{(a-l/2)} + \frac{1}{(a+l/2)}\right]$$

$$= k\lambda\left[\frac{-(a+l/2)+(a-l/2)}{\left(a^2-\frac{l^2}{4}\right)}\right]$$

$$E = \frac{k\lambda l}{(a^2-l^2/4)} \text{ along OP} \quad (2)$$

In the problem

$$\lambda = \frac{q}{l} = -\frac{22\times10^{-6}}{0.4} = -5.5\times10^{-5} \text{ C/m}.$$

$\therefore$ From 2,
$$E = \frac{(9.0\times10^{9})(-5.5\times10^{-5})(0.4)}{[(0.36)^2-(0.2)^2]}$$

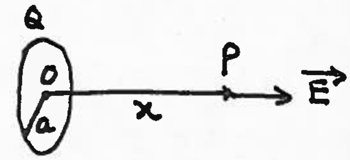
$$= -\frac{1.98\times10^{5}}{8.96\times10^{-2}} = -2.21\times10^{6} \text{ N/C (along OP)}$$

or $= 2.21\times10^{6}$ N/C along PO

Q15 A uniformly charged ring of radius 10 cm has a total charge of 75 μC. Find the electric field on the axis of the ring at (a) 1.0 cm, (b) 5.0 cm, and (c) 100 cm from the center of the ring.

(6.65×10^6 N/C, 2.42×10^7 N/C, 6.65×10^5 N/C)

<u>Solution:</u> Field at a point P at a distance x on the axis is given by



$$E = \frac{kxQ}{(x^2+a^2)^{3/2}} \text{ along the axis}$$

(a) radius of ring → $a = 10$ cm $= 0.1$ m

Charge on the ring → $Q = 75\,\mu C = 75 \times 10^{-6}$ C

Distance of P from the Center → $x = 1$ cm $= 0.01$ m.

$$\therefore E = \frac{(9\times10^9)(0.01)(75\times10^{-6})}{[(.01)^2 + (.1)^2]^{3/2}}$$

$$= \frac{6.75\times10^3}{(1.01\times10^{-2})^{3/2}} = 6.65\times10^6 \text{ N/C (along OP)}$$

(b) when $x = 5$ cm $= 0.05$ m.

$$E = \frac{(9\times10^9)(0.05)(75\times10^{-6})}{[(0.05)^2 + (0.1)^2]^{3/2}} = \frac{3.38\times10^4}{(1.25\times10^{-2})^{3/2}}$$

$$= 2.42 \times 10^7 \text{ N/C (along OP)}$$

(c) when $x = 100$ cm. $= 1$ m

$$E = \frac{(9\times10^9)(1)(75\times10^{-6})}{[1+(.1)^2]^{3/2}} = \frac{6.75\times10^5}{(1.01)^{3/2}}$$

$$= 6.65\times10^5 \text{ N/C (along OP)}$$

Note: In (b) field **E** is stronger at 5 cm than at 1 cm in (a)