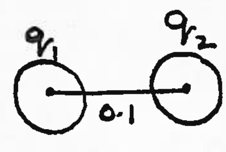
Solutions
Assignment 1
P. 205/2

Q1 Each of two small spheres is positively charged, the combined charge totaling 4×10^{-8}C. What is the charge on each sphere if they are repelled with a force of 27×10^{-5}N when place 0.1 m apart? (3.0×10^{-8}C, 1.0×10^{-8}C)

Solution: If q_1 and q_2 are the charges on the two spheres, the force between them is



$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 (0.1)^2} = 27 \times 10^{-5} \text{ (given)}$$

$$\text{or } q_1 q_2 = 4\pi\epsilon_0 (0.1)^2 \times 27 \times 10^{-5}$$

$$q_1 q_2 = \frac{(0.1)^2 \times 27 \times 10^{-5}}{9 \times 10^9} = 3 \times 10^{-16} \quad (1) \qquad \left[\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{N.m^2}{C^2}\right]$$

$$q_1 + q_2 = 4 \times 10^{-8} \quad (2)$$

From (1) and (2), $q_1 + \frac{3 \times 10^{-16}}{q_1} = 4 \times 10^{-8}$

$$\text{or } q_1^2 + 3 \times 10^{-16} = 4 \times 10^{-8} q_1$$

$$\text{or } q_1^2 - 4 \times 10^{-8} q_1 + 3 \times 10^{-16} = 0$$

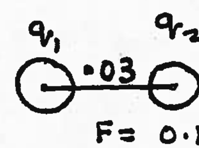
$$\text{or } (q_1 - 3 \times 10^{-8})(q_1 - 1 \times 10^{-8}) = 0$$

$\therefore$ The two charges are 3×10^{-8}C and 1×10^{-8}C

Q2 Two small spheres carrying unequal positive charges repel each other with a force of 0.12 N when 3 cm apart. If the charge on each sphere is doubled and the distance between them is tripled, what is the force of repulsion? (ans: 0.056 N)

Solution: $r = 0.03m$, $F = 0.12 N$

$\therefore$ using $\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}$,



$$0.12 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{(0.03)^2} \quad (1)$$

If the charges on the spheres are doubled and the distance tripled,

$$F' = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2q_1)(2q_2)}{(3 \times 0.03)^2} = \frac{4}{9}\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{(0.03)^2}\right) \quad (2)$$

From (1), $F' = 4/9 \times 0.12 = 0.053 N$ Ans

Q3

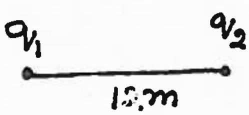
Charges of +5 and +10 μC and 12m apart. What is the force between them? (ans: 0.45N)

Solution:

$q_1 = 5\mu C = 5\times10^{-6} C$

$q_2 = 10\mu C = 10\times10^{-6} C$

$r = 12 m$



$\therefore \quad F = \frac{1}{(4\pi\epsilon_0)} \frac{q_1 q_2}{r^2} =$ $\qquad \frac{1}{4\pi\epsilon_0} = 9.0\times10^9 \ N.m^2$

$= (9.0\times10^9)\frac{(5\times10^{-6})(10\times10^{-6})}{(12)^2} = 0.45 N$

Q4

How far apart in a vacuum must two electron be if the force of electrostic repulsion on each electron is just equal in magnitude to the weight of electron?. (ans: 5.09 m)

Solution:

Let r be the distance between two electrons

From Coulomb's Law: $F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ (1)

$q_1 = q_2 = e = 1.6\times10^{-19} C$, $1/4\pi\epsilon_0 = 9.0\times10^{9} \frac{N.m^2}{C^2}$

weight of electron $= (9.11\times10^{-31})\times 9.8$ N

$\therefore \quad 9.11\times10^{-31}\times 9.8 = \frac{9.0\times10^9 (1.6\times10^{-19})^2}{r^2}$

$\therefore \quad \frac{1}{r} = \sqrt{\frac{(9.11\times10^{-31})(9.8)}{(9.0\times10^9)(1.6\times10^{-19})^2}} = 0.197$

$\therefore$ distance between two electrons = 5.08 m.

Q.5

Two charges of +15 and +5 μC are 6 cm apart, where must a third charge be placed in order that the resultant force acting on it be zero? (ans: 3.8 cm)

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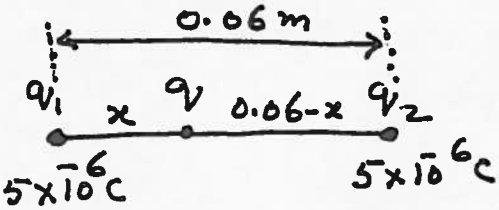
Q.5 **Solution:**



If x is the distance of the third charge from q_1, and q is the charge on it. To have resultant force to be zero on it, the forces on q due to q_1, q_2 must be equal in magnitude but opposite in direction.

$$\therefore \left(\frac{1}{4\pi\epsilon_0}\right)\frac{(15\times10^{-6}).q}{x^2} = \frac{\left(\frac{1}{4\pi\epsilon_0}\right)(5\times10^{-6})q}{(0.06-x)^2}$$

$$\therefore \frac{3}{x^2} = \frac{1}{(0.06-x)^2}$$

or $3(0.06-x)^2 = x^2$

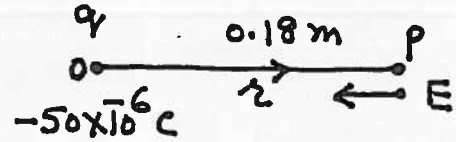
or $\sqrt{3}(0.06-x) = x$ or $x = \dfrac{\sqrt{3}\times 0.06}{(1+\sqrt{3})} = 0.038\,m$.

$\therefore$ The distance of third charge from q_1 = 3.8 Cm.

Q.6

What are the magnitude and direction of the electric intensity at a point 18 cm from a point charge of –50 μC? (ans: -1.39×10^7 N/C)

Solution: The electric field due to a charge at a point P, at a distance r from q, is given by



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}\hat{r}$$, $\hat{r}$ is a unit vector along OP

Given: $q = 50\times10^{-6}$C

$r = 18$ Cm $= 0.18\,m$.

$\left(\frac{1}{4\pi\epsilon_0}\right) = 9.0\times10^9 \frac{N.m^2}{C^2}$ for Vacuum.

$$\therefore \vec{E} = \frac{(9.0\times10^9)(-50\times10^{-6})}{(0.18)^2} = -1.39\times10^7\,\frac{N}{C}$$

The direction of $\vec{E}$ is along PO

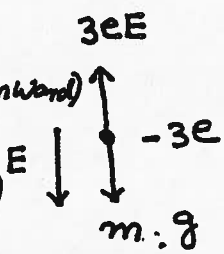
Q7 An oil drop has a net negative charge of –3e, representing an excess of three electrons. It remains at rest under the action of the force of gravity and the electric force when it is placed in a downward electric field of 3.0×10^5 N/C. Find the mass of the droplet. (ans: 1.47×10^{-14} kg)

Solution:

Charge on the drop $= -3e = -3 \times 1.6 \times 10^{-19}$ C

Force of gravity (weight) $= m\,g$ (downward)

($m \rightarrow$ mass of the drop) $= m(9.8)$ N (1)



Electrical force on the drop $= 3eE$ (upward)

$= 3(1.6 \times 10^{-19})(3.0 \times 10^{5})$ N (2)

Since the drop remains at rest, the two forces in (1) and (2) must be equal.

$$\therefore \quad m \times 9.8 = 3 \times 1.6 \times 10^{-19} \times 3.0 \times 10^{5}$$

$$\text{or } m = \frac{(3 \times 1.6 \times 10^{-19})(3.0 \times 10^{5})}{9.8} = 1.47 \times 10^{-14} \text{ kg.}$$

Q8 What is the acceleration of an electron in a field of 5×10^5 N/C? Express this in terms of the acceleration of gravity. (ans: 8.8×10^{16} m/s^2, 8.95×10^{15} g)

Solution: The acceleration of the electron is due to the electrical force in the field $E = 5 \times 10^5$ N/C.

If a is the acceleration of the electron, then from Newton's second law of motion

$$m_e a = F$$ where m_e = mass of electron

But $F = eE$ where e = charge of electron, E = magnitude of electric field

$$\therefore a = \frac{F}{m_e} = \frac{eE}{m_e}$$

$$= \frac{(1.6 \times 10^{-19})(5 \times 10^{5})}{(9.11 \times 10^{-31})} = 8.8 \times 10^{16} \text{ m/s}^2$$

$$= \frac{8.8 \times 10^{16}}{9.8} = 8.95 \times 10^{15} g.$$

Q9 $6.02\text{x}10^{23}$ atoms of monatomic hydrogen have a mass of one gram. How far would the electron of a hydrogen atom have to be removed from the nucleus for the force of attraction to equal the weight of the atom. (ans: $1.19\text{x}10^{-1}$ m)

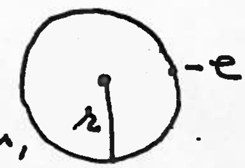
Solution: No. of atoms in 1 gram of hydrogen $= 6.02\times10^{23}$

$\therefore$ weight of one atom of hydrogen $= \dfrac{0.001}{6.02\times10^{23}} = 1.66\times10^{-27}$ kg.

$= (1.66\times10^{-27})(9.8)$ N

The electrostatic force on the electron

$$F_e = \frac{(1.6\times10^{-19})(1.6\times10^{-19})}{4\pi\epsilon_0 r^2}$$



For the force to be equal to the weight of the atom,

$$\frac{(1.6\times10^{-19})^2}{4\pi\epsilon_0 r^2} = (1.66\times10^{-27})(9.8)$$

or $$r = \sqrt{\frac{(1.6\times10^{-19})^2}{4\pi\epsilon_0(1.66\times10^{-27})(9.8)}} = \sqrt{\frac{(9.0\times10^{9})(1.6\times10^{-19})^2}{1.66\times9.8\times10^{-27}}}$$

$$= 1.19\times10^{-1}\,m$$

Q10 If all the positive charges in a molecule of hydrogen atoms were lumped into a single charge, and all the negative charges into a single charge, what force would the two lumped charges exert on each other at a distance of (a) 1 m, (b) 10^7 m. (ans: $8.35\text{x}10^{19}$ N, $8.35\text{x}10^{5}$ N)

Solution: No of atoms of hydrogen in 1 gram of H atom $= 6.02\times10^{23}$

Each H-atom has one proton and one electron

$\therefore$ Total positive charge in one gram-mole of H

$$= (6.02\times10^{23})\times(1.6\times10^{-19})$$
$$= 9.63\times10^{4}\,c$$

Total negative charge is also $= -9.63\times10^{4}\,c$

(a) when $r = 1\,m$, Force between them, $F = \dfrac{(9\times10^{9})(9.63\times10^{4})^2}{1^2}$

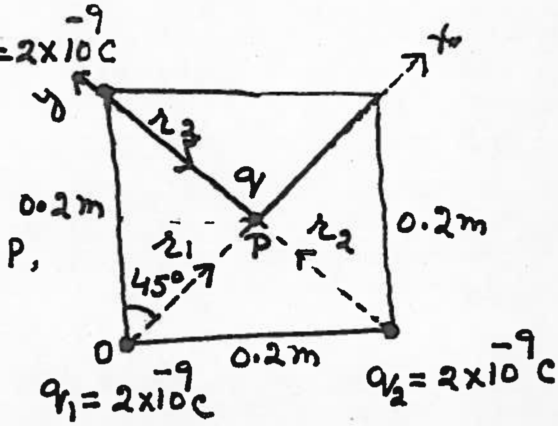
$$= 8.35\times10^{19}\,N$$

(b) when $r = 10^{7}\,m$, $F = \dfrac{(9\times10^{9})\times(9.63\times10^{4})^2}{(10^{7})^2} = 8.35\times10^{5}\,N$

In both Cases, the force is attractive

Q 11 Point charges of 2×10^{-9} C are situated at each of the three corners of a square whose side is 0.20 m. What would be the magnitude and direction of the resultant force on a point charge of -1×10^{-9} C if it were placed (a) at the center of the square? (b) at the vacant corner of the square

<u>Solution</u>



(a) $r_1 = r_2 = r_3 = 0.2 \cos 45°$
$= 0.141$ m

Force, F_1 on a charge $(-1\times10^{-9}$c$)$ at P,

$\vec{F_1} = F_{1x}\hat{i} + F_{1y}\hat{j}$ where $\hat{i}, \hat{j}$ are unit vectors along x, y axes as shown.

$$F_{1x} = \frac{(9\times10^{9})(2\times10^{-9})(-1\times10^{-9})}{(0.141)^2} = -9.05$$

$F_{1y} = 0$

$\therefore \vec{F_1} = -9.05\times10^{-7}\hat{i} + 0\hat{j}$

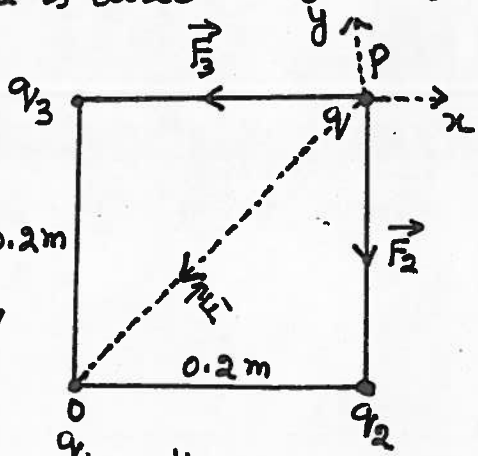
Similarly,

$\vec{F_2} = F_{2x}\hat{i} + F_{2y}\hat{j} = 0\hat{i} + (-9.05\times10^{-7})\hat{j}$

and $\vec{F_3} = F_{3x}\hat{i} + F_{3y}\hat{j} = 0\hat{i} + (9.05\times10^{-7})\hat{j}$

$\therefore$ Resultant $\rightarrow R = \vec{F_1} + \vec{F_2} + \vec{F_3} = -9.05\times10^{-7}\hat{i} + 0\hat{j}$

The resultant force is attractive, and is directed towards q_1 along the line PO.



(b) $q_1 = q_2 = q_3 = 2\times10^{-9}$c

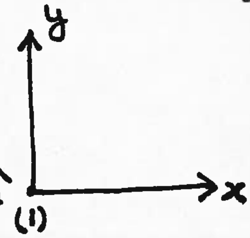
Charge at P $= -1\times10^{-9}$c.

$\vec{F_1} = \hat{i}F_{1x} + \hat{j}F_{2y}$, $OP = \sqrt{(.2)^2 + (.2)^2} = 0.283$

$$F_{1x} = \frac{(9\times10^{9})(2\times10^{-9})(-1\times10^{-9})\cos 45°}{(0.283)^2} = -1.59\times10^{-7}$$

$$F_{2y} = \frac{(9\times10^{9})(2\times10^{-9})(-1\times10^{-9})\sin 45°}{(0.283)^2}$$

$= -1.59\times10^{-7}$



$\therefore \vec{F_1} = F_{1x}\hat{i} + F_{2y}\hat{j} = -1.59\times10^{-7}\hat{i} - 1.59\times10^{-7}\hat{j}$ (1)

$\vec{F_2} \rightarrow$ Force due to q_2 on $q = F_{2x}\hat{i} + F_{2y}\hat{j}$

$F_{2x} = 0$, $F_{2y} = (9\times10^{9})(2\times10^{-9})(-1\times10^{-9}) = -4.5\times10^{-7}$

$\therefore \vec{F_2} = 0\hat{i} - 4.5\times10^{-7}\hat{j}$

Similarly,

$\vec{F_3} = F_{3x}\hat{i} + F_{3y}\hat{j} = -4.5\times10^{-7}\hat{i} + 0\hat{j}$

$\therefore$ Resultant $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

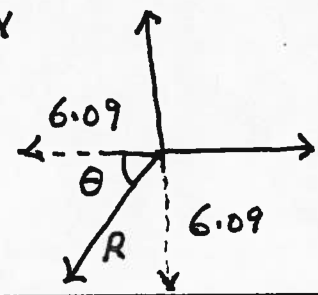
$= (-1.59\times10^{-7}\hat{i} - 1.59\times10^{-7}\hat{j}) + (0\hat{i} - 4.5\times10^{-7}\hat{j}) + (-4.5\times10^{-7}\hat{i} + 0\hat{j})$

$\vec{R} = (-6.09\times10^{-7})\hat{i} - (6.09\times10^{-7})\hat{j}$

Magnitude of $\vec{R} \rightarrow R = \sqrt{(-6.09\times10^{-7})^2 + (-6.09\times10^{-7})^2}$

$= 8.61\times10^{-7} N$

Direction of R is given by



$\tan\theta = \frac{-6.09}{-6.09} = 1$

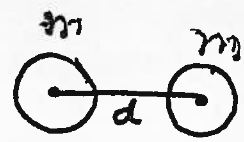
or $\theta = 45^{0}$

Q12 A certain metal sphere of volume 1 cm^3 has a mass of 7.5g and contains 8.2×10^{22} electrons. (a) How many electrons must be removed from each of two such spheres so that the electrostatic force of repulsion between them just balances the force of gravitational attraction? (treat the charges on the spheres as point charges), (b) express the number of electrons removed as a fraction of the total number of free electrons. (ans: 4.03×10^{6}, 4.92×10^{-17})

Solution

(a) mass of each sphere $\rightarrow m = 7.5\ g$

$= 7.5\times10^{-3}\ kg.$



Gravitational force, $F_g = \frac{G m^2}{d^2} = \frac{(6.67\times10^{-11})(7.5\times10^{-3})^2}{d^2}$

Electrostatic repulsion between the two spheres is,

$F_e = \frac{(9\times10^{9})\, q^2}{d^2}$

If $F_g = F_e$, $\frac{(6.67\times10^{-11})(7.5\times10^{-3})^2}{d^2} = \frac{(9\times10^{9})\, q^2}{d^2}$

$\therefore q = \sqrt{\frac{(6.67\times10^{-11})(7.5\times10^{-3})^2}{9\times10^{9}}} = 6.457\times10^{-13} C$

(Continued ----)

Number of electrons which must be removed to attain this charge is,

$$N = \frac{q}{e} = \frac{6.457\times10^{-13}}{1.6\times10^{-19}} = 4.03\times10^{6}$$

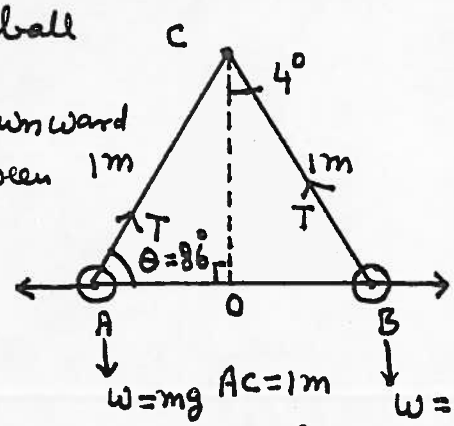
(b) Fraction, $f = \frac{\text{Number of electrons removed}}{\text{Total number of free electrons}}$

$$= \frac{4.03\times10^{6}}{8.2\times10^{22}} = 4.92\times10^{-17}$$

Q13

Two small balls, each of mass 10g, are attached to silk threads 1m long and hung from a common point. When the balls are given equal quantities of negative charge, each thread makes an angle 0f 4° with the vertical. (a) Draw a diagram showing all the forces on each ball, (b) Find the magnitude of the charge on each ball. (ans: 1.22×10^{-7}C)

Solution: Forces acting on each ball are:
- T along the silk thread
- W weight of the ball acting downward
- F electrostatic repulsion between the balls.



AC = 1m

AO = AC Cos θ

= AC Cos 86° = 0.07 m

∴ AB = 0.14m

Considering the equilibrium of the ball A,

$$T\cos 86^\circ - F = 0 \quad (1)$$

$$T\sin 86^\circ - W = 0 \quad (2)$$

But $W = mg = 0.01\times9.8 = 0.098N$

and $F = \frac{(9.0\times10^{9})\,q^2}{(0.14)^2} = 4.59\times10^{11}q^2 \quad (3)$

From (2), $T = \frac{W}{\sin 86^\circ} = \frac{0.098}{0.998} = 0.1N \quad (4)$

From (1), $F = T\cos 86^\circ = 0.1\times0.07 = 0.007N \quad (5)$

From (3) and (5), $q^2 = \frac{F}{4.59\times10^{11}} = \frac{0.007}{4.59\times10^{11}} = 1.53\times10^{-14}$

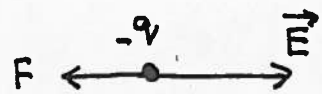
$\therefore q = 1.24\times10^{-7}C.$

Q14

A botany professor is in an electric field. The uniform electric field has a magnitude of 500 N/C. An electrical force of magnitude 6.0 N is found to act on the professor of mass 75 kg in the direction opposite to that of the field. What is the charge of the professor? (ans: -1.2×10^{-2} C)

Solution: Force $\vec{F}$ on a charge q in an electric field $\vec{E}$ is given by, $\vec{F} = q\vec{E}$

Since the direction of $\vec{F}$ on the professor is opposite to the direction of the field, the charge on the professor must be negative.



The magnitude of the force is

$$F = qE = 6\,N$$

$$q = \frac{6}{E} = \frac{6}{500} = 1.2\times10^{-2}\,C$$

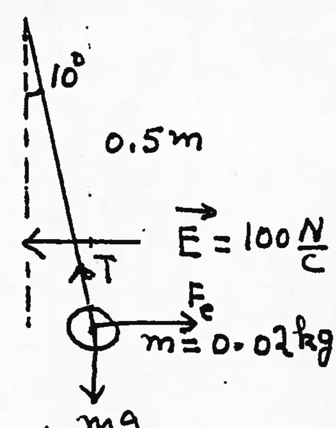
Since the charge is negative, $q = -1.2\times10^{-2}$

Q.15

A small charged sphere of mass 20.0 gram is suspended at rest making 10° angle to the right of the vertical, by means of a thin (massless) thread of length 50.0 cm in a uniform electric field of magnitude 100 N/C directed in the horizontal direction to the left. (a) What is the sign of the charge on the sphere? (b) Find the charge on the sphere. (ans: -ve, -3.5×10^{-4} C)

Solution:

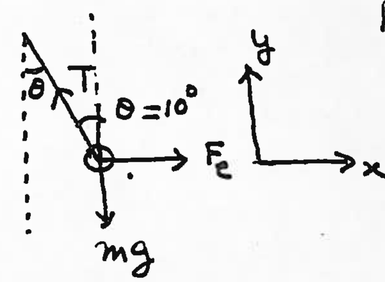
(a) Since the sphere is being deflected in a direction opposite to the direction of electric field, the charge on it must be. negative



(b) The forces acting on the small charged sphere are:

1. Its weight $W = mg$ directed downward
2. The tension $\vec{T}$ of the string, directed along the string and away from the sphere
(3) The electrostatic force $\vec{F_e}$ directed horizontally to the right.

using Conditions of equilibrium in horizontal and vertical direction,



$$-T\cos 80^{\circ} + F_e = 0 \quad (1)$$

$$T\sin 80^{\circ} - mg = 0 \quad (2)$$

$$mg = 0.02 \times 9.8 = 0.196\ N$$

From (2) $\quad T = \dfrac{mg}{\sin 80^{0}} = \dfrac{0.196}{0.985} = 0.199\ N \quad (3)$

The electrostatic force, $F_e = qE = 100q \quad (4)$

From (1), $\quad F_e = T\cos 80^{0} = 0.199 \times 0.174 = 0.035 N$

From 4 $\quad \therefore\ q = \dfrac{F_e}{100} = \dfrac{0.035}{100} = 3.5 \times 10^{-4} C$

Since the charge is negative, $\quad q = -3.5 \times 10^{-4} C$

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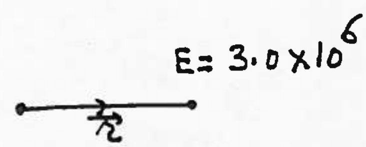
At what distance from a proton is the magnitude of its electric field equal to 3.0×10^{6} N/C? (ans: 2.2×10^{-8} m)

Solution: Charge on proton $= 1.6 \times 10^{-19} c$.

The magnitude of electric field is given by,



$$E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2} \quad (1)$$

$E = 3.0 \times 10^{6}\ N/c$, $\quad \therefore\ 3.0 \times 10^{6} = \dfrac{(9.0 \times 10^{9})\, q}{r^2} \quad (2)$

$q \rightarrow$ charge on proton $= 1.6 \times 10^{-19} c$.

$\therefore$ From (2)

$$r^2 = \frac{(9 \times 10^{9})(1.6 \times 10^{-19})}{(3.0 \times 10^{6})} = 4.8 \times 10^{-16}\ m^2$$

$$\therefore r = \sqrt{4.8 \times 10^{-16}} = 2.2 \times 10^{-8} m.$$

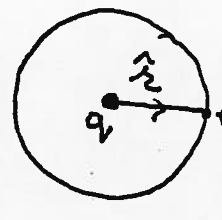
Q 17

Assume the electron in the ground state of the hydrogen atom is located 5.29×10^{-11} m from the nuclear proton. (a) what is the magnitude of the electric field of the proton at the position of the electron? (b) What is the magnitude of the electrical force on the electron? (c) What is the magnitude of the electric field of the electron at the position of the proton? (d) What is the magnitude of the electric force on the proton? (ans: 5.15×10^{11} N/C, 8.25×10^{-8} N, 5.15×10^{11} N/C, 8.25×10^{-8} N)

Solution:

$r = 5.29\times10^{-11}\,m$

$q \rightarrow$ Charge on proton $= 1.6\times10^{-19}\,C$



(a) magnitude of electric field at the position of e is,

$$E_p = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2} = \frac{(9.0\times10^{9})(1.6\times10^{-19})}{(5.29\times10^{-11})^2} = 5.15\times10^{11}\,N/C$$

(b) magnitude of electric force on e is given by,

$$F_e = qE = (-1.6\times10^{-19})(5.15\times10^{11})$$

$$= 8.24\times10^{-8}\,N$$

(c) Magnitude of electric field at q due to electron is.

$$E_e = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2} = \frac{(9.0\times10^{9})(-1.6\times10^{-19})}{(5.29\times10^{-11})^2}$$

$$= 5.15\times10^{11}\,N/C\,(\hat{r})$$

(d) Magnitude of electric force on proton is

$$\vec{F}_p = q\vec{E} = (1.6\times10^{-19})(5.15\times10^{11})$$