University of Calgary

Faculty of Science Midterm test

PHYSICS 259 ALL LECTURE SECTIONS

February 13, 2018, 7:00-9:00 pm Time: 120 minutes

This is a closed-book exam worth a total of 32 points. Please answer all questions.

ONLY THE FOLLOWING ITEMS ARE ALLOWED ON YOUR DESK DURING THE EXAM:

1. THIS MULTIPLE-CHOICE QUESTION BOOKLET, which includes the multiple-choice exam questions and the formula sheet (last page). THIS BOOKLET WILL **NOT** BE HANDED IN. Make sure this booklet contains 16 pages (including formula sheet). If you are missing any pages, get a new booklet from the exam supervisor.

2. A BUBBLE SHEET used to answer multiple-choice questions. **IT WILL BE HANDED IN**.

IMPORTANT: START by entering your ID NUMBER, NAME and COURSE ID on the bubble sheet. Using a pen or a pencil, black out the corresponding numbers below your ID and name. DO THIS NOW. Then wait for the Exam Supervisor to signal when to start the test.

All answers to the multiple-choice questions must be entered by blacking out the appropriate character (one of a, b, c, d) beside the question number on the bubble sheet. Make sure you darken the entire interior of the circle that contains the character.

3. A CALCULATOR, which can be anything that does NOT connect to wifi and does NOT communicate with other devices. ACCEPTABLE calculators are only Schulich School of Engineering calculators. UNACCEPTABLE calculators include: programmable calculators, cell phones, tablets, laptops, etc.

4. A PEN OR PENCIL, used to black out your answers on the bubble sheet. If you are using a PEN, make sure it is BLACK or BLUE in order to ensure the scanner reads your answers properly. PENCILS can also be used, but make sure you press firmly when answering so the scanner reads your answer properly.

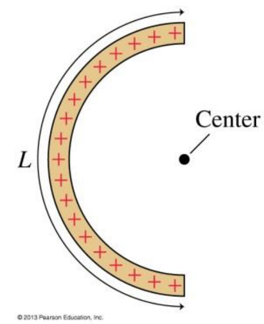
5. YOUR STUDENT ID CARD, used to check your identity during exam sign-in.

If you are missing anything from the above items, raise your hand and ask an exam supervisor to supply what is missing. If you are missing an item that should have been brought by you (e.g., calculator, pen/pencil) there is a limited supply of extras and are on a first come, first served basis.

Please note that you can leave the room at any time after the first 30 minutes (after the signing-in process have been completed) and before 90 minutes of the exam. <u>Students remaining after 90 minutes must remain seated for the duration of the exam until permitted to leave by the exam supervisor.</u>

Multiple Choice Questions (Total: 32 marks)

1. A uniformly charged insulating rod is bent in the shape of a semi-circle of radius R. The total charge on the rod is $+Q$. There are no other charges nearby. Which of the following best describes the magnitude E of the electric field at the center of the semi-circle?



a) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$
b) $E < \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$
c) $E > \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$
d) $E = 0$

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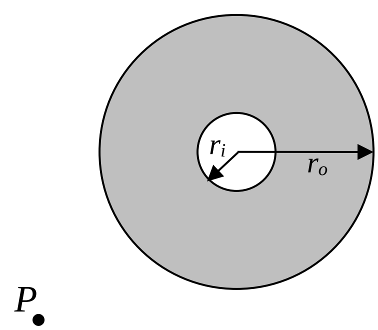
2. An insulating spherical shell of inner radius r_i = 3.00 cm and outer radius r_o = 8.00 cm is charged with a uniform volume charge density $\rho = 6.0\times10^{-15}$ C/cm^3. What is the magnitude of the electric field at point P, a distance 16.0 cm from the center of the spherical shell?



a) E = 4.9 N/C
b) E = 5.4 N/C
c) E = 5.8 N/C
d) E = 4.3 N/C

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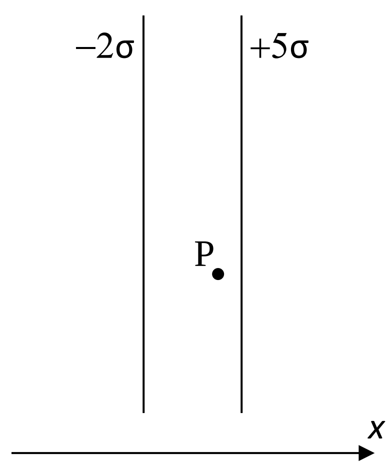
3. Two very thin infinite sheets are uniformly charged with surface charge densities -2σ and $+5\sigma$ as indicated in the figure. What is the x-component of the electric field at point P located between the sheets?



a) $+3\sigma/2\varepsilon_0$
b) $+7\sigma/2\varepsilon_0$
c) $-3\sigma/2\varepsilon_0$
d) $-7\sigma/2\varepsilon_0$

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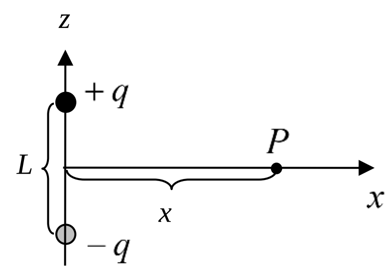
4. Two charges, $+q$ and $-q$ separated by a distance L, form a dipole as shown in the figure. Consider a point a distance x along the perpendicular bisector of the line joining the two charges. If x is much larger than L, how does the magnitude of the electric field vary with x?
Hint: the exact expressions for the electric field in this case is:

$$\vec{E}_{dipole} = \frac{-\vec{p}}{4\pi\varepsilon_o\,(x^2 + L^2/4)^{3/2}}$$



a) The magnitude of the electric field is proportional to x^{-3}.
b) The magnitude of the electric field is proportional to x^{-2}.
c) The magnitude of the electric field is proportional to x^{-1}.
d) The magnitude of the electric field is constant.

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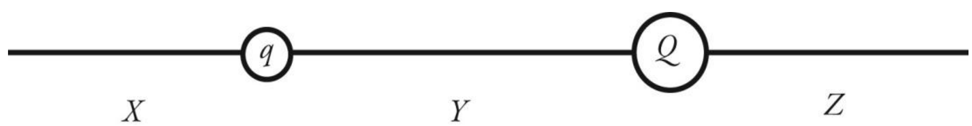
5. The figure below shows two unequal point charges, q and Q, of opposite sign. Charge Q has greater magnitude than charge q. In which of the regions X, Y, Z (defined by the horizontal line that passes through the centers of the charges), could one place a positive test charge that will experience zero net force from the other two charges?



a) only regions X and Z
b) only region Y
c) only region X
d) only region Z

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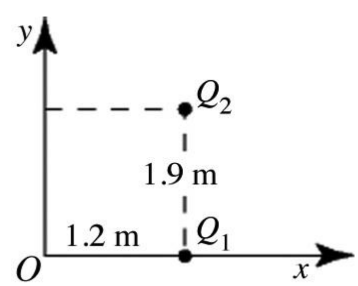
6. Two point charges, $Q_1 = -1.0\ \mu C$ and $Q_2 = +4.0\ \mu C$, are placed as shown in the figure. What is the y-component of the electric field at the origin O?



a) $+6.0\times10^3$ N/C
b) $+3.8\times10^3$ N/C
c) -6.0×10^3 N/C
d) -3.8×10^3 N/C

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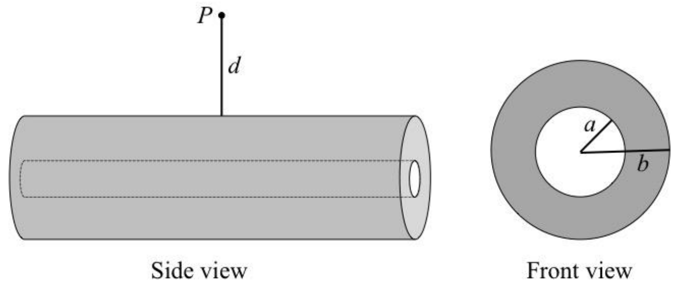
7. A long thick cylindrical insulating tube of inner radius a and outer radius b carries a uniform volume charge density ρ, as shown in the diagram. What is the electric field strength at point P a distance d from the surface of the tube? Assume that $L >> d$.



a) $E = \frac{\rho b^2}{2\epsilon_0 d}$
b) $E = \frac{\rho}{2\pi\epsilon_0 (b+d)}$
c) $E = \frac{\rho(b^2-a^2)}{2\pi\epsilon_0 d}$
d) $E = \frac{\rho(b^2-a^2)}{2\epsilon_0 (b+d)}$

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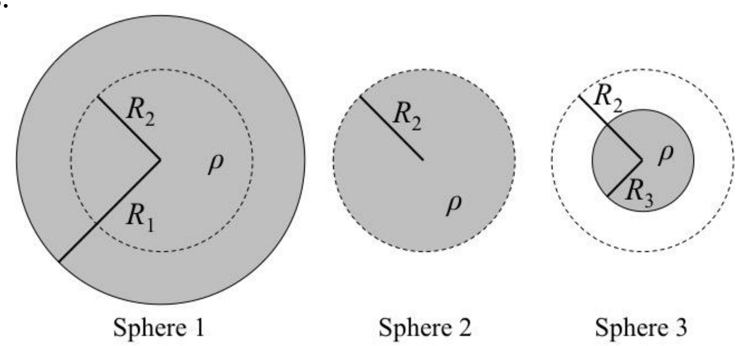
8. Three charged insulating spheres of radii R_1, R_2, and R_3 all carry the same volume charge density ρ. A Gaussian surface of radius R_2 is placed concentric with each of the spheres, as shown in the diagram below. Rank the electric field strengths on the Gaussian surfaces for the three spheres.



a) $E_1 > E_2 > E_3$
b) $E_1 = E_2 > E_3$
c) $E_2 > E_1 > E_3$
d) $E_1 > E_2 = E_3$

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9. Which of the following is a correct statement of Gauss's Law?

a) Electric field lines always begin on positive charges and end on negative charges.
b) The electric force between two point charges is given by the Coulomb force.
c) The magnitude of the electric field is proportional to the density of electric field lines.
d) The net electric flux through a closed surface is proportional to the charge enclosed.

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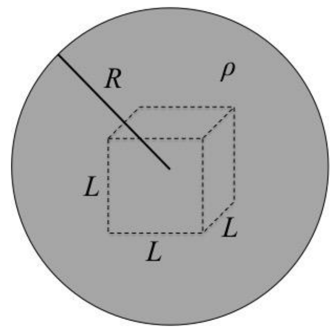
10. A charged insulating sphere has uniform volume charge density ρ = 2.53 μC/m^3 and radius R = 53.4 cm. A Gaussian cube of side length L = 37.8 cm is placed inside the sphere. What is the electric flux through the Gaussian cube?



a) 1.08×10^5 N m^2/C
b) 1.54×10^4 N m^2/C
c) 1.82×10^5 N m^2/C
d) Gauss's Law does not apply here.

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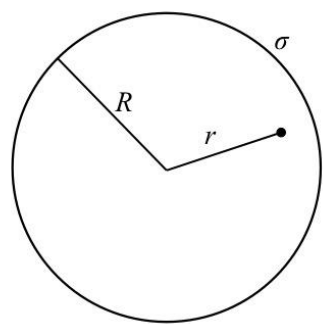
11. A thin spherical insulating shell of radius R = 10.0 cm carries a uniform surface charge density σ = 1.50 μC/m^2. What is the electric field strength at a distance r = 8.00 cm from the centre of the shell?



a) 1.69×10^5 N/C
b) 2.65×10^5 N/C
c) 2.11×10^6 N/C
d) 0.00 N/C

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12. A region of space contains a uniform electric field $\vec{E} = -(3.00\ \mathrm{N/C})\hat{\jmath}$. What is the electric flux through a circular loop whose diameter is D = 1.00 m and whose unit normal vector is given by $\hat{n} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$

a) –2.12 N m^2/C
b) –1.67 N m^2/C
c) –6.66 N m^2/C
d) –5.30 N m^2/C

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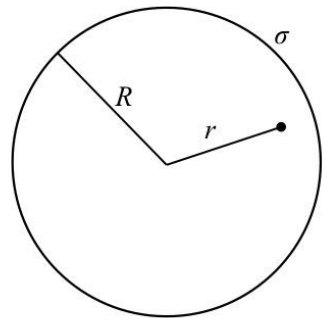
13. A thin spherical insulating shell of radius $R = 1.00$ m carries a uniform surface charge density $\sigma = 3.56\ \mu\text{C/m}^2$. What is the electric potential at a distance $r = 89.0$ cm from the centre of the shell? Take the zero of potential to be at infinity.



a) 0.00 V
b) 4.52×10^5 V
c) 3.58×10^5 V
d) 4.02×10^5 V

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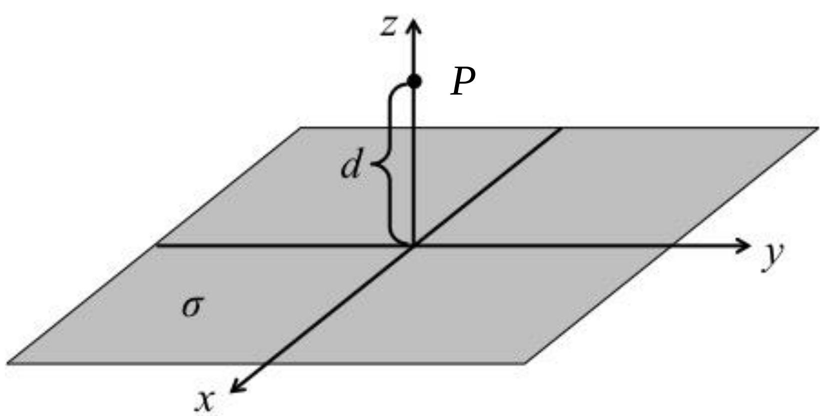
14. An infinite sheet of charge in the x,y-plane sitting at $z = 0$ has a uniform surface charge density $\sigma = -9.00\ \text{nC/m}^2$. Choosing the zero of potential to be at $z = 0$, what is the electric potential energy of a proton at point P a distance $d = 2.75$ cm above the sheet?



a) $+2.24\times10^{-18}$ J
b) $+4.47\times10^{-18}$ J
c) -4.47×10^{-18} J
d) -2.24×10^{-18} J

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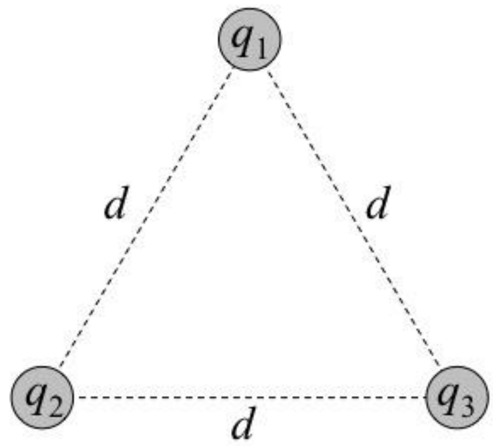
15. Three charges, $q_1 = +1.0\ \mu C$, $q_2 = +2.0\ \mu C$, and $q_3 = –3.0\ \mu C$, are arranged in an equilateral triangle of edge length $d = 1.0$ m. What is the electric potential energy of this configuration of charges?



a) –62.9 mJ
b) –98.9 mJ
c) –8.99 mJ
d) –45.0 mJ

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16. Which of the following statements regarding electric potential is true?

a) The electric potential increases in the direction perpendicular to the electric field.
b) The electric potential is always defined to be zero at infinity.
c) The electric potential decreases in the direction of the electric field.
d) The electric potential is another name for electric potential energy.

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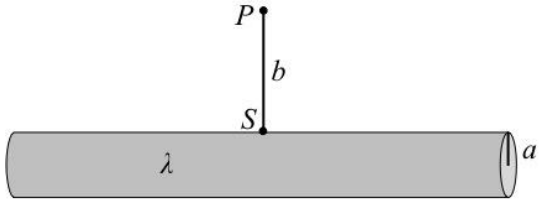
17. A very long insulating cylinder of radius a carries a uniform linear charge density λ. What is the potential difference $\Delta V = V_P - V_S$ between point S on the surface of the cylinder and point P located a distance b away from the surface?



a) $\Delta V = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$
b) $\Delta V = \frac{\lambda}{2\pi\epsilon_0(a+b)}\ln\left(\frac{b+a}{a}\right)$
c) $\Delta V = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b+a}{a}\right)$
d) $\Delta V = -\frac{\lambda}{2\pi\epsilon_0 a}\ln\left(\frac{a}{b}\right)$

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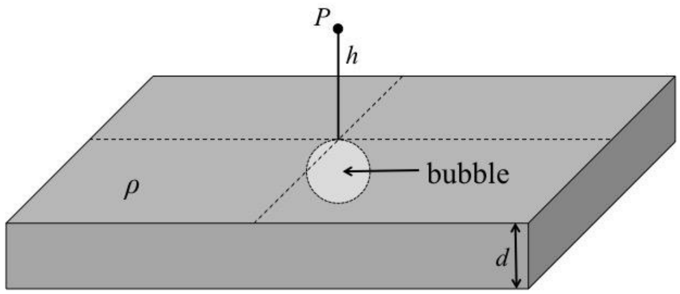
18. In a classical model of a helium ion (He^+), the nucleus is approximated as a point charge $+2e$ and an electron is in circular orbit at a distance of 0.26×10^{-10} m from the nucleus. What is the magnitude of the force on the electron due to the nucleus?

a) 3.4×10^{-7} N
b) 6.8×10^{-7} N
c) 1.4×10^{-6} N
d) 2.9×10^{-6} N

a

Questions 19, 20 and 21 are all related to the following situation:

A very large slab of insulating material has a thickness d = 5.0 cm and is uniformly charged throughout its volume with charge density ρ = 6.8 nC/m^3. The slab contains a spherical air bubble whose diameter is exactly the thickness of the slab d. Point P is located a height h = 1.0 cm above the top of the air bubble. Assume the length and width of the slab are much greater than the distance h.



19. Which of the following statements regarding this situation is true?

a) This object can be treated as the superposition of a solid slab with charge density $+\rho$ and a solid sphere with charge density $-\rho$.
b) At point P, this object can be treated as an infinite plane with surface charge density $\sigma = \rho d$.
c) The electric field above this object is uniform, provided that h is much smaller than the length and width of the slab.
d) The presence of the bubble means that we <u>cannot</u> use Gauss's Law to find the electric field at point P.

20. If the slab above were solid (*i.e.*, if the air bubble was not there), what would be the effective surface charge density σ, treating the slab as an equivalent plane of charge?

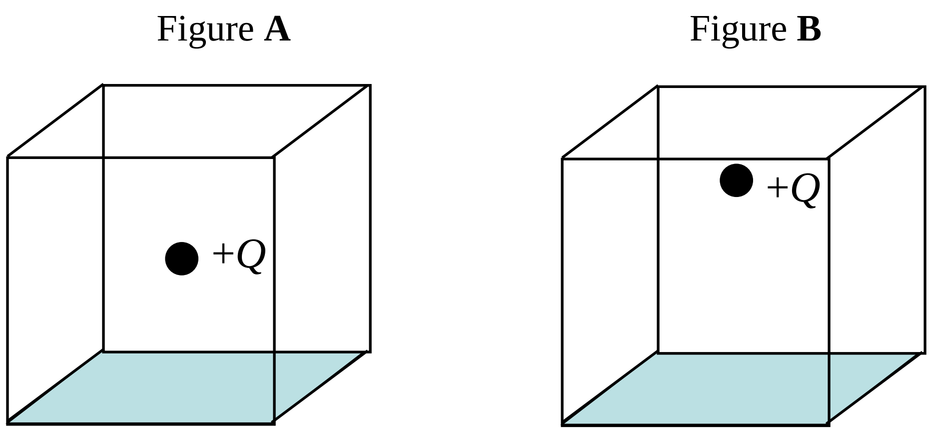
a) 0.34 nC/m^2
b) 0.68 nC/m^2
c) 1.0 nC/m^2
d) 6.8 nC/m^2

21. What is the electric field strength E at point P for the slab with a hole shown above?

a) 15.9 N/C
b) 17.6 N/C
c) 19.2 N/C
d) 20.8 N/C

Questions 22 and 23 are all related to the following situation:

In Figure **A**, a positive charge $+Q$ is placed at the center of a cube. In Figure **B**, this same positive charge is moved upwards, closer to the upper surface of the cube.



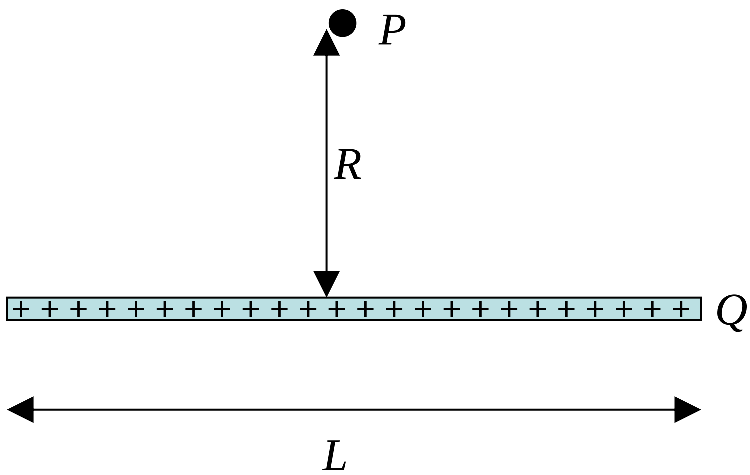
22. Consider the total electric flux Φ_{cube} passing outward through the cube:

a) Φ_{cube} in Figure **A** > Φ_{cube} in Figure **B**.
b) Φ_{cube} in Figure **A** < Φ_{cube} in Figure **B**.
c) Φ_{cube} is the same in both figures.
d) It cannot be determined from the information given.

23. Consider the electric flux Φ_{bottom} passing outward through the shaded bottom face of the cube:

a) Φ_{bottom} in Figure **A** > Φ_{bottom} in Figure **B**.
b) Φ_{bottom} in Figure **A** < Φ_{bottom} in Figure **B**.
c) Φ_{bottom} is the same in both figures.
d) It cannot be determined from the information given.

24. A thin insulating rod of length L has a charge Q spread uniformly along it. Point P is at a distance R from the middle point of the rod, as shown in the figure below. What is the magnitude of the electric field, E, at point P?



Hint: Do not try to derive the solution. Instead, try to select the correct answer by considering the limiting cases.

a) $E = \frac{Q}{2\pi \varepsilon_0 R} \frac{1}{(L^2+4R^2)^{\frac{3}{2}}}$
b) $E = \frac{Q}{2\pi \varepsilon_0 R} \frac{1}{(L^2+4R^2)^{\frac{1}{2}}}$
c) $E = \frac{Q}{2\pi \varepsilon_0 R} \frac{1}{(4L^2+R^2)^{\frac{1}{2}}}$
d) $E = \frac{Q}{2\pi \varepsilon_0 R} \frac{1}{(4L^2+R^2)^{3/2}}$

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25. A dipole with dipole moment $\vec{p}$ is immersed in a uniform electric field $\vec{E}$. Initially the angle between $\vec{p}$ and $\vec{E}$ is $\theta_i = 45°$. An external agent does work on the dipole and changes the angle to $\theta_f = 135°$. How does the potential energy of the dipole in the final state compare to that in the initial state?

a) $U_f = -2U_i$
b) $U_f = -1U_i$
c) $U_f = +1U_i$
d) $U_f = +2U_i$

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26. The permanent electric dipole moment of a water molecule is 6.2×10^{-30} C m. What is the maximum possible torque on a water molecule in a uniform electric field of magnitude 5.0×10^{8} N/C?

a) 6.2×10^{-30} N m
b) 8.1×10^{37} N m
c) 1.2×10^{-38} N m
d) 3.1×10^{-21} N m

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27. The electric field 5.0 cm from a very long charged wire is 2000 N/C, directed toward the wire. What is the charge on a 1.0 cm long segment of the wire?

a) -5.6×10^{-11} N/C
b) -2.8×10^{-11} N/C
c) $+5.6\times10^{-11}$ N/C
d) $+2.8\times10^{-11}$ N/C

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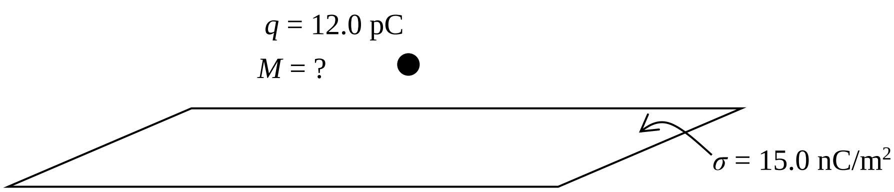
28. A small dust particle is given a net positive charge q = +12.0 pC (1 pC = 10^{-12} C). It is positioned and floats above a very large sheet (approximated as an infinite plane), with a surface charge density of σ = +15.0 nC/m^2. What is the particle's mass?



a) 1.50×10^{-19} kg
b) 16.5×10^{-12} kg
c) 1.04×10^{-9} kg
d) 1.65×10^{-6} kg

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29. Two protons (p_1 and p_2) are on the *x*-axis, as shown below. The directions of the electric field at points 1, 2, and 3, respectively, are:

a) →, ←, →
b) ←, →, ←
c) ←, →, →
d) ←, ←, →

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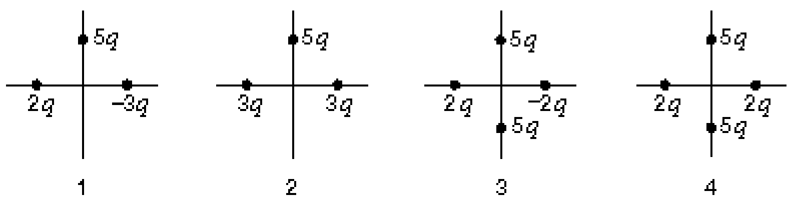
30. The diagrams below depict four different charge distributions. The charged particles are all the same distance from the origin. The electric field at the origin:

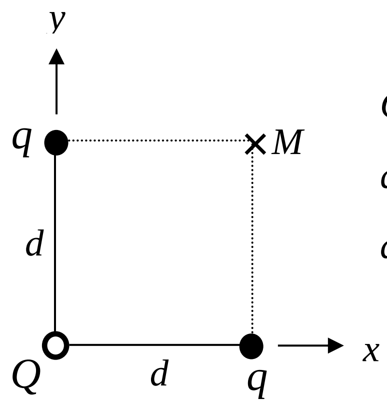


a) is least for situation 1
b) is greatest for situation 3
c) is zero for situation 4
d) is downward for situation 1

Questions 31 and 32 are all related to the following situation:

Three point charges are fixed in place in the *xy*-plane at the positions shown in the figure below. The point *M* is located at $(x,y) = (d,d)$, also in the *xy*-plane.

b



$Q = -8.0\ \mu C$

$q = +3.0\ \mu C$

$d = 5.0$ cm

31. What is the *x*-component of the electric field at the point *M*?

a) $E_x = +2.1\times10^7$ N/C
b) $E_x = -3.6\times10^6$ N/C
c) $E_x = +6.2\times10^5$ N/C
d) $E_x = -1.8\times10^7$ N/C

32. If a charge $Q' = 2.0\ \mu C$ having mass $m = 0.20$ kg is placed at the point *M* and then released from rest (with the other three charges remaining fixed in place), what speed will it have when it gets very far away from the other three charges?

a) $v = 1.1$ m/s
b) $v = 2.7$ m/s
c) $v = 4.2$ m/s
d) $v = 6.5$ m/s

Formulae and Constants

Electrostatics		
$\vec{F}_e = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$	$\vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$	$\vec{F}_e = q\vec{E}$
$U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$	$V = k\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$	$U = qV$
$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \oiint E\,dA\cos\theta = \frac{Q_{enc}}{\varepsilon_0}$		$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$
$\Delta V = -\int_a^b \vec{E}\cdot\vec{dl}$	$W = -q\Delta V$	$C = \frac{\varepsilon_0 A}{d}$
$C = \kappa C_0 = \kappa\varepsilon_0\frac{A}{d} = \varepsilon\frac{A}{d}$	$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$	$u = \frac{1}{2}\varepsilon_0 E^2$
$\vec{p} = qs$, from $-$ to $+$	$\vec{\tau} = \vec{p}\times\vec{E}$	$U = -\vec{p}\cdot\vec{E}$
Electrodynamics		
$\Delta V_R = IR$	$P = IV = I^2R = \frac{V^2}{R}$	$\vec{J} = \sum_i n_i q_i \vec{v}_i$
$R = R_1 + R_2 + R_3 + \cdots$	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$	$R = \frac{\rho L}{A}$
$\Delta V_C = \frac{Q}{C}$	$C = C_1 + C_2 + C_3 + \cdots$	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
$\sum_{\text{junction}} i = 0$	$\sum_{\text{loop}} (\mathcal{E} + \Delta V_R + \Delta V_C) = 0$	$\tau = RC$
Magnetostatics		
$\vec{F}_m = q\vec{v}\times\vec{B}$	$\Phi_B = \int \vec{B}\cdot d\vec{A}$	$\vec{F} = I\vec{l}\times\vec{B}$
$\vec{\mu} = I\vec{A}$	$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$	$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$	$\vec{\tau} = \vec{\mu}\times\vec{B}$	$U = -\vec{\mu}\cdot\vec{B}$
$nq = -\frac{J_x B_y}{E_z}$	$r = \frac{mv}{qB}$	
Magnetodynamics		
$\mathcal{E} = -\frac{d\Phi_B}{dt}$	$\mathcal{E} = \int_a^b (\vec{v}\times\vec{B})\cdot d\vec{l}$	$\mathcal{E}_2 = -N_2\frac{d\Phi_{B2}}{dt}$
$\mathcal{E}_2 = -M\frac{di_1}{dt}$	$M = \frac{N_2\Phi_2}{i_1} = \frac{N_1\Phi_1}{i_2}$	$\mathcal{E} = -L\frac{dI}{dt}$
$L = \frac{N\Phi}{i}$	$U = \frac{1}{2}LI^2$	$u = \frac{1}{2\mu_0}B^2$
$\tau = \frac{L}{R}$	$x = x_0 e^{-t/\tau}$	$x = x_0\left(1 - e^{-t/\tau}\right)$

Formulae and Constants (continued)

Fundamental Constants		
$k = 8.99 \cdot 10^{9}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$	$\varepsilon_0 = 8.85 \cdot 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$	$\mu_0 = 4\pi \cdot 10^{-7}\ \frac{\mathrm{T \cdot m}}{\mathrm{A}}$
$q_e = -1.602 \cdot 10^{-19}\ \mathrm{C}$	$m_e = 9.11 \cdot 10^{-31}\ \mathrm{kg}$	$m_p = 1.67 \cdot 10^{-27}\ \mathrm{kg}$
$g = 9.81\ \mathrm{N/kg}$		

Kinematics and Dynamics			
$\sum \vec{F} = m\vec{a}$	$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x$

Mathematical Formulae & Prefixes			
milli $(m) = 10^{-3}$	micro $(\mu) = 10^{-6}$	nano $(n) = 10^{-9}$	pico $(p) = 10^{-12}$

$C = 2\pi r$	$A_{CIRCLE} = \pi r^2$	$A_{SPHERE} = 4\pi r^2$
$V_{SPHERE} = \frac{4}{3}\pi r^3$	$A_{CYL} = 2\pi r L$	$V_{CYL} = \pi r^2 L$

$\int \frac{dx}{x} = \ln x$	$\int x^n dx = \frac{1}{n+1} x^{n+1}$
$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$	$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a}$
$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$	$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \quad if\ x \ll 1, then\ (1 + x)^n \cong 1 + nx$$