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Assignment #1 – **ERRORS**

Clearly explain what you are doing.
Print legibly or type
Place the answer to every question in a box
Use a straight edge.

1. Suppose the numbers $x_1, x_2, \ldots, x_n$ are approximations to $X_1, X_2, \ldots, X_n$ and that in each case the maximum possible error is E.
   a) Prove that the maximum possible error in the sum of x_i is nE.

Since

$$x_i - E \le X_i \le x_i + E$$

it follows by the addition that

$$\sum x_i - nE \le \sum X_i \le \sum x_i + nE$$

So that

$$-nE \le \sum X_i - \sum x_i \le nE$$

Which is what was to be proved.

   b) Compute the sum $\sqrt{1} + \sqrt{2} + \cdots + \sqrt{100}$ with all the roots evaluated to two decimal places. By the preceding problem, what is the maximum possible error?

Whether by a few well-chosen lines of programming or by a more old-fashioned appeal to tables, the roots in question can be found and summed. The result is 671.38.

Each square root has two decimal places. Therefore all the digits after 0.0A… ($1 \le A \le 9$ will be chopped to 0.0A. The maximum possible amount of chop-off error is 0.0099999…, which can be approximated to 0.01. For the round-off the maximum possible error is the half of this amount which is 0.005.
Chop-off:
Each root has a maximum error of E=.01, the maximum possible error in the sum is nE= 100(.01) =1.0
Round-off
Each root has a maximum error of E=.005, the maximum possible error in the sum is nE= 100(.005) =0.5,

The Maclaurin series expansion for cos x is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\cdots$$

Starting with the simplest version, estimate cos($\pi/3$) by adding terms one at a time. After each new term is added, calculate the true and approximate percent relative errors. Consider just four terms.

a) Use a calculator to determine the "true" value., and add terms until the absolute value of the approximate error estimate falls below an error criterion conforming to two significant digits.

b) Repeat using only three significant figures (after conducting each individual mathematical operation and chop=off to three significant figures).
c) What observations can be made from these calculations?

SOLUTION

a) **True value via calculator**

$$\varepsilon_s = 0.5\times 10^{2-2}\% = 0.5\%$$

True value: cos(π/3) = 0.5

zero order:

$$\cos\left(\frac{\pi}{3}\right) = 1$$

$$\varepsilon_t = \left|\frac{0.5-1}{0.5}\right|\times 100\% = 100\%$$

first order:

$$\cos\left(\frac{\pi}{3}\right) = 1 - \frac{(\pi/3)^2}{2} = 0.451689$$

$$\varepsilon_t = 9.66\% \qquad \varepsilon_a = \left|\frac{0.451689-1}{0.451689}\right|\times 100\% = 121.4\%$$

second order:

$$\cos\left(\frac{\pi}{3}\right) = 0.451689 + \frac{(\pi/3)^4}{24} = 0.501796$$

$$\varepsilon_t = 0.359\% \qquad \varepsilon_a = \left|\frac{0.501796-0.451689}{0.501796}\right|\times 100\% = 9.986\%$$

third order:

$$\cos\left(\frac{\pi}{3}\right) = 0.501796 - \frac{(\pi/3)^6}{720} = 0.499965$$

$$\varepsilon_t = 0.00709\% \qquad \varepsilon_a = \left|\frac{0.499965 - 0.501796}{0.499965}\right| \times 100\% = 0.366\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

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b) <u>Repeat using only three significant figures (after conducting each individual mathematical operation and chop=off to three significant figures).</u>

<u>Note:</u> all the calculations in grey column involve the calculations using the entries above in the column and then truncation to three digits using the Excel command :Trunc".t

Estimate at x = (π/3) =	1.047198		COS (π/3) =	0.5
	All Digits	Chop-off to 3 figures		
Zerot order Maclaurin	1	1		
First order term				
(π/3)	1.047198	1.047		
$(\pi/3)^2$	1.096623	1.096		
2!	2	2		
$(\pi/3)^2$ / 2!	0.548311	0.548		
Substract from zero order				
First order Maclaurin	0.451689	0.452		
ε_t =		9.60		
ε_a =		-121.24		
Second order term				
(π/3)	1.047198	1.047		
$(\pi/3)^4$	1.202581	1.201		
4!	24	24		
$(\pi/3)^4$ / 4!	0.050108	0.05		
Add to first order				
Second Order Maclaurin	0.501689	0.502		
ε_t =		-0.40		
ε_a =		9.96		

$$\varepsilon_t = \left|\frac{0.5 - 1}{0.5}\right| \times 100\% = 100\%$$

$$\varepsilon_t = \left|\frac{0.5 - 0.452}{0.5}\right| \times 100\% = 9.60\%$$

$$\varepsilon_a = \left|\frac{0.452 - 1}{0.452}\right| \times 100\% = 121.24\%$$

$$\varepsilon_t = \left|\frac{0.5 - 0.502}{0.5}\right| \times 100\% = 0.40\%$$

$$\varepsilon_a = \left|\frac{0.502 - 0.452}{0.502}\right| \times 100\% = 9.96\%$$

Third order term		
(π/3)	1.047198	1.047
$(\pi/3)^6$	1.318778	1.317
6!	720	720
$(\pi/3)^6$ / 6!	0.001829	0.001
Substract from second order		
Third Order Maclaurin	0.499859	0.501
ε_t =		-0.20
ε_a =		-0.20

$$\varepsilon_t = \left|\frac{0.5-0.501}{0.5}\right| \times 100\% = 0.20\%$$

$$\varepsilon_a = \left|\frac{0.501-0.502}{0.501}\right| \times 100\% = 0.20\%$$

Fourth order term		
(π/3)	1.047198	1.047
$(\pi/3)^8$	1.446202	1.444
8!	40320	40320
$(\pi/3)^8$ / 8!	3.58E-05	0
Add to third order		
Fourth Order Maclaurin	0.499824	0.501
ε_t =		-0.20
ε_a =		0.00

$$\varepsilon_t = \left|\frac{0.5-0.501}{0.5}\right| \times 100\% = 0.20\%$$

$$\varepsilon_a = \left|\frac{0.501-0.501}{0.501}\right| \times 100\% = 0.0\%$$

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c)What observations can be made from these calculations?

When only three digits are used

a) the predictions differ slightly and are not as accurate.

The fourth and higher order approximations do no yield an improvement in the predictions because we are dealing with the addition of a small number to one that is much larger and the contribution of the smaller number gets lost because the three digit cut-off

2. As will see later on in your fluid mechanics course, the headloss caused by water traveling in a pipe (full of water) can be described by the Darcy-Weisbach equation:

$$h = \lambda \frac{L}{D}\frac{V^2}{2g}$$

where λ = headloss coefficient (unitless) ;
V= average flow velocity (m/s);
L = length of the pipe (m);
h = headloss (m);
D = diameter of the pipe (m);
g = gravitational acceleration (m/s^2);

The values of the relevant variables and the uncertainty (error in the measurement) in them are
$L = 400 \pm 8$,
$D = 0.8 \pm 0.125$,
$V = 2 \pm 0.04$.

The values of the other model coefficients are:

$$\lambda = 0.015$$
$$g = 10$$

a- Estimate the uncertainty in the values of h.

b- The uncertainty of which input variable (L,D or V) has the greatest impact on the value of the pressure drop (h)? Why?

(a) First, calculate the pressure drop by ignoring the uncertainty:

$$h = (0.015)\frac{400}{0.8}\frac{2^2}{2 \cdot 10} = 1.5m$$

To determine the uncertainty, calculate the contribution to the error from each variable, ie:

$$dh = \left|\frac{\partial h}{\partial L}\right| dL + \left|\frac{\partial h}{\partial D}\right| dD + \left|\frac{\partial h}{\partial V}\right| dV$$

Partial derivatives are:

$$\frac{\partial h}{\partial L} = \lambda \frac{V^2}{2Dg} = \frac{h}{L}$$
$$\frac{\partial h}{\partial D} = -\lambda \frac{LV^2}{2D^2 g} = -\frac{h}{D}$$
$$\frac{\partial h}{\partial V} = \lambda \frac{VL}{Dg} = \frac{2h}{V}$$

And so the uncertainty is given by:

$$\Delta h = h\left(\left|\frac{\Delta L}{L}\right| + \left|-\frac{\Delta D}{D}\right| + \left|\frac{2\Delta V}{V}\right|\right)$$
$$\Delta h = 1.5\left(\frac{8}{400} + \frac{0.125}{0.8} + \frac{2 \cdot 0.04}{2}\right) = 0.324375 \approx 0.324$$
$$h \in [1.176, 1.824]$$

Finally, the pressure drop range is h ± Δh:
If we consider the source of the uncertainty we notice that:

$$dh = \left|\frac{\partial h}{\partial L}\right| dL + \left|\frac{\partial h}{\partial D}\right| dD + \left|\frac{\partial h}{\partial V}\right| dV = h\left(\left|\frac{\Delta L}{L}\right| + \left|-\frac{\Delta D}{D}\right| + \left|\frac{2\Delta V}{V}\right|\right)$$

it is noted that for a proportional uncertainty, the velocity of the fluid will have twice the impact of the other parameters. It is therefore concluded that the velocity of the fluid has the greatest impact on the uncertainty of the pressure drop.

4. Submit your work for Task 8 of Lab #2. Note that this involves two slight modifications:
   a) The drag force now equals $F_D = c \cdot V^2$
   b) The mass is dependent on your student number.

Submit a copy of the VBA as well as the copy of the spreadsheet with your answer

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Solution

$$\sum F = mg - cv^2 \rightarrow ma = mg - cv^2 \rightarrow m\frac{dv}{dt} = mg - cv^2 \rightarrow \frac{dv}{dt} = g - \frac{cv^2}{m} \rightarrow \Delta v = \left(g - \frac{cv^2}{m}\right)\Delta t \rightarrow$$

$$v_2(t_{i+1}) - v_1(t_i) = \left(g - \frac{c\ v^2(t_i)}{m}\right)\Delta t$$

Where:

$\Delta t = 1$

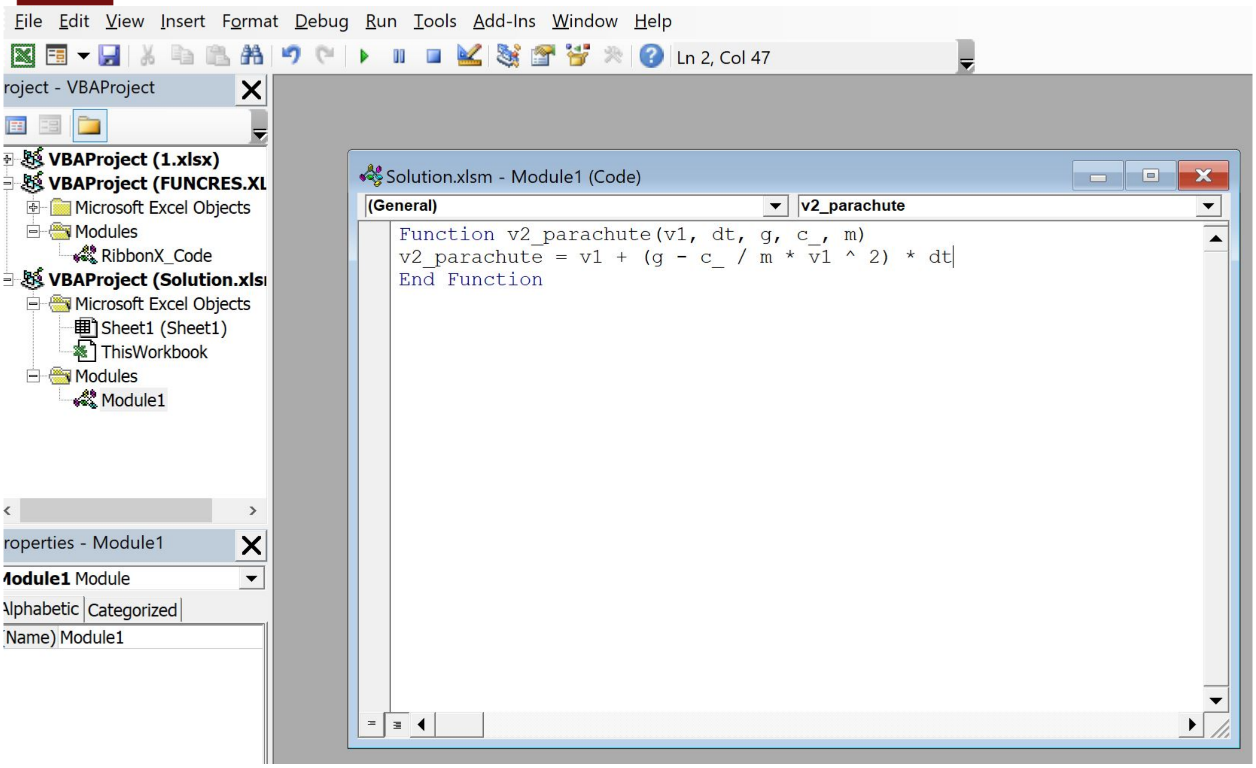
$c = 0.125\,\frac{kg}{s}$

$g = 9.81\,\frac{m}{s^2}$

$$m = 35\left(1 + \frac{S\# - 19 \times 10^5}{31 \times 10^5}\right)kg = 35\left(1 + \frac{8377295 - 19 \times 10^5}{31 \times 10^5}\right) = 108.13$$

By having the highlighted equation and the constants we can find v_2 values.

First, it is needed to insert the codes and define the function like below:



Then by calling v2_parachute function, v2 values are obtained which can be plotted as a function of t.

The initial value of v is 0 (D3).