Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

MAT1330 D: Calculus for life sciences I

Instructor: Aziz Khanchi

Test II - Ver. 0
March 2011

Surname ______________________ First Name ______________________

Student # ______________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

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- Where it is possible to check your work, do so.
- Good Luck!

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Q1	Q2	Q3	Q4	Q5	Total
/6	/7	/4	/6	/17	/40

Question 1. [6 points] Compute the derivatives of the following functions. Do NOT simplify.

(a) $f(x) = \ln\left(e^{5x^2}(x+2)^3\right)$.

$$f'(x) = \frac{1}{e^{5x^2}(x+2)^3}\left[10x\,e^{5x^2}(x+2)^3 + e^{5x^2}\left(3(x+2)^2\right)\right]$$

$f'(x) =$ $\boxed{}$

(b) $g(x) = \sqrt{\sin(x^2)}$. $= (\sin x^2)^{1/2}$

$$g'(x) = \frac{1}{2}(\sin x^2)^{-1/2}(\cos x^2)(2x)$$

$g'(x) =$ $\boxed{}$

Question 2. Consider the DTDS $x_{t+1} = 0.5(x_t)^2 + hx_t$ where h is a nonnegative parameter.

(a) [2 points] Find an equilibrium point of the system as a function of h.

$$x^* = 0.5 x^{*2} + h x^* \xrightarrow{x^* \neq 0} 1 = 0.5 x^* + h \longrightarrow$$

$$\frac{1-h}{0.5} = x^* \longrightarrow x^* = 2(1-h)$$

Answer: $x^* =$ $\boxed{2(1-h)}$

(b) [1 point] Determine the interval for h where x^* is positive. Remember that $h \geq 0$.

$$2(1-h) \geqslant 0 \Rightarrow (1-h) \geqslant 0 \Rightarrow 1 \geqslant h$$

Answer: $\boxed{0 \leqslant h \leqslant 1}$

(c) [2 points] Define the function $P(h) = hx^*$ where x^* is the equilibrium in part (a). Find h that maximizes $P(h)$ on the interval in part (b).

$$P(h) = 2h(1-h) = 2(h-h^2) \longrightarrow P'(h) = 2(1-2h) = 0$$

$$\longrightarrow h = 0.5$$

h	$P(h)$	
0.5	0.5	← max value
0	0	
1	0	

Answer: $h =$ $\boxed{0.5}$

Question 3. Consider the function $f(x) = \cos(x) - x$.

(a) [2 points] Explain why this function has a zero (or root) in the interval $[0, \pi/2]$.

$\cos x - x$ is continuous on $[0, \pi/2]$ and $f(0) = 1 - 0 > 0 > f(\frac{\pi}{2}) = \cos\frac{\pi}{2} - \frac{\pi}{2} = \frac{-\pi}{2}$. Using Intermediate Value Thm $f(x)$ has a root between 0 and $\frac{\pi}{2}$.

(b) [2 points] Use the Rolle's Theorem to show that f has only one root in the interval $(0, \pi/2)$.

Suppose that f has two roots a and b, then $f(a) = f(b) = 0$. The Rolle's Thm implies that c exists between a and b s.t.

$f'(c) = 0 \longrightarrow -\sin c - 1 = 0 \longrightarrow \sin c = -1$

But this can't happen on $(0, \frac{\pi}{2})$.

Question 4. Consider the DTDS

$$x_{t+1} = \frac{1 + x_t^2}{1 + x_t}, \qquad t = 0, 1, 2, \ldots.$$

(a) [1 point] The updating function of this DTDS is $f(x) =$ $\boxed{\frac{1+x^2}{1+x}}$

(b) [2 points] The only positive equilibrium point is $x^* =$ $\boxed{1}$

$$x^* = \frac{1+x^{*2}}{1+x^*} \longrightarrow x^* + x^{*2} = 1 + x^{*2} \Longrightarrow x^* = 1$$

(c) [2 points] According to the derivative test, is the steady state stable or unstable?

Answer: $\boxed{\text{stable}}$

$$f'(x) = \frac{2x(1+x) - (1+x)}{(1+x)^2}$$

$$f'(1) = \frac{4 - 2}{4} = \frac{1}{2} \qquad |f'(1)| < 1 \longrightarrow 1 \text{ is stable}$$

(d) [1 point] Starting from $x_0 = 2$ calculate x_1, x_2, x_3. Answer: $\boxed{x_1 = 1.67,\ x_2 = 1.42,\ x_3 = 1.25}$

$$x_1 = \frac{1+4}{3} = \frac{5}{3} = 1.67 \qquad x_2 = \frac{1+1.67^2}{1+1.67} = 1.42$$

$$x_3 = \frac{1+1.42^2}{1+1.42} = 1.25$$

Question 5. Consider the function $f(x) = xe^{-x}$ on the interval $[0, \infty)$. Follow these steps to graph the function.

(a) [1 point] The domain of f is $\boxed{[0, \infty)}$

(b) [2 points] The derivative of f is $f' = \boxed{e^{-x}(1-x)}$

$$f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$$

(c) [2 points] The critical point(s) of f are $\boxed{1}$

$$f'(x) = 0 \longrightarrow e^{-x}(1-x) = 0 \longrightarrow x = 1$$

(d) [3 points] The function is increasing on the interval $\boxed{(0,1)}$

The function is decreasing on the interval $\boxed{(1, \infty)}$

x		1	
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$

(e) [2 points] The second derivative of f is $f'' =$ $\boxed{e^{-x}(x-2)}$

$$f''(x) = -e^{-x}(1-x) - e^{-x} = e^{-x}(-1+x-1) = e^{-x}(-2+x)$$

(f) [3 points] The function is concave upward on the interval $\boxed{(2, \infty)}$

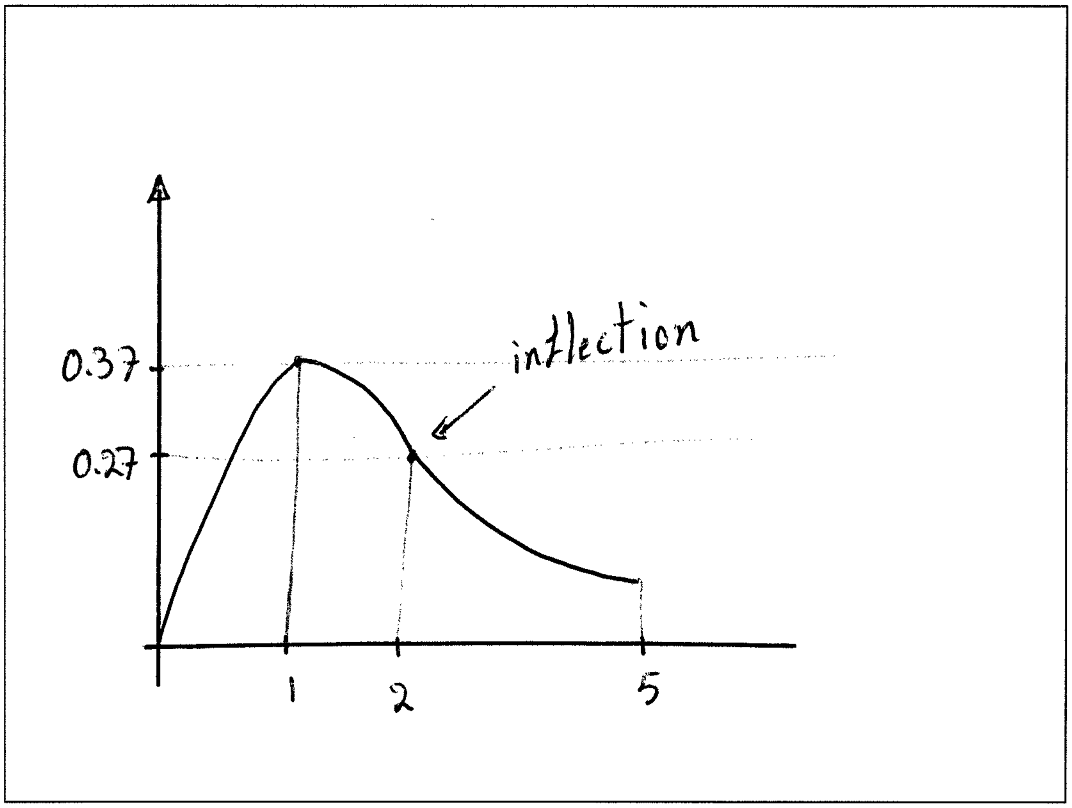
The function is concave downward on the interval $\boxed{(0, 2)}$

x		2	
$f''(x)$	$-$	0	$+$
$f(x)$	$\cap$		$\cup$

(g) [1 point] The point(s) of inflection of f are $\boxed{2.}$

$f(x)$ changes concavity at $x = 2$.

(h) [3 points] The graph of f for $x \in [0, 5]$ is



$f(1) = e^{-1} = 0.37$

$f(2) = 2e^{-2} = 0.27$

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Question 1. [6 points] Compute the derivatives of the following functions. Do NOT simplify.

(a) $f(x) = \ln\left(e^{5x^3}(x+2)^4\right)$.

$$f'(x) = \frac{1}{e^{5x^3}(x+2)^4}\left[15x^2 e^{5x^3}(x+2)^4 + e^{5x^3}(4(x+2)^3)\right]$$

$f'(x) =$ $\boxed{}$

(b) $g(x) = \sqrt{\sin(x^2)}$.

$$g'(x) = \frac{1}{2}(\sin x^2)^{-1/2}(\cos x^2)(2x)$$

$g'(x) =$ $\boxed{}$

Question 2. Consider the DTDS $x_{t+1} = 0.25(x_t)^2 + hx_t$ where h is a nonnegative parameter.

(a) [2 points] Find an equilibrium point of the system as a function of h.

$$x^* = 0.25\, x^{*2} + h x^* \xrightarrow{x^* \neq 0} 1 = 0.25\, x^* + h \longrightarrow$$

$$x^* = \frac{1}{0.25}(1-h) = 4(1-h)$$

Answer: $x^* = \boxed{4(1-h)}$

(b) [1 point] Determine the interval for h where x^* is positive. Remember that $h \geq 0$.

$$4(1-h) \geqslant 0 \Longrightarrow h \leqslant 1$$

Answer: $\boxed{0 \leqslant h \leqslant 1}$

(c) [2 points] Define the function $P(h) = hx^*$ where x^* is the equilibrium in part (a). Find h that maximizes $P(h)$ on the interval in part (b).

$$P(h) = 4h(1-h) = 4(h - h^2)$$

$$P'(h) = 4(1-2h) = 0 \Longrightarrow h = 0.5$$

h	$P(h)$
0.5	1 ← max
0	0
1	0

Answer: $h = \boxed{0.5}$

Question 3. Consider the function $f(x) = \cos(x) - x$.

(a) [2 points] Explain why this function has a zero (or root) in the interval $[0, \pi/2]$.

(b) [2 points] Use the Rolle's Theorem to show that f has only one root in the interval $(0, \pi/2)$.

Question 4. Consider the DTDS

$$x_{t+1} = \frac{2 + x_t^2}{1 + x_t}, \qquad t = 0, 1, 2, \ldots.$$

(a) [1 point] The updating function of this DTDS is $f(x) = \boxed{\frac{2+x^2}{1+x}}$

(b) [2 points] The only positive equilibrium point is $x^* = \boxed{2}$

$$x^* = \frac{2+x^{*2}}{1+x^*} \longrightarrow x^* + x^{*2} = 2 + x^{*2} \longrightarrow x^* = 2$$

(c) [2 points] According to the derivative test, is the steady state stable or unstable?

Answer: $\boxed{\text{Stable}}$

$$f'(x) = \frac{2x(1+x) - (2+x^2)}{(1+x)^2}$$

$$f'(2) = \frac{(4)(3) - (2+4)}{(1+2)^2} = \frac{12-6}{9} = \frac{6}{9} \qquad |f'(2)| < 1 \longrightarrow$$

2 is stable

(d) [1 point] Starting from $x_0 = 2$ calculate x_1, x_2, x_3. Answer: $\boxed{x_1 = 2,\ x_2 = 2,\ x_3 = 2}$

$$x_1 = \frac{2+4}{1+2} = \frac{6}{3} = 2, \quad x_2 = 2, \quad x_3 = 2$$

Question 5. Consider the function $f(x) = xe^{-x}$ on the interval $[0, \infty)$. Follow these steps to graph the function.

(a) [1 point] The domain of f is $\boxed{}$

(b) [2 points] The derivative of f is $f' = \boxed{}$

(c) [2 points] The critical point(s) of f are $\boxed{}$

(d) [3 points] The function is increasing on the interval $\boxed{}$

The function is decreasing on the interval $\boxed{}$

(e) [2 points] The second derivative of f is $f'' =$

(f) [3 points] The function is concave upward on the interval

The function is concave downward on the interval

(g) [1 point] The point(s) of inflection of f are

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$f'(x) =$ $\boxed{}$

(b) $g(x) = \sqrt{\sin(x^2)}$.

$$g'(x) = \frac{1}{2}(\sin x^2)^{-1/2}(\cos x^2)(2x)$$

$g'(x) =$ $\boxed{}$

Question 2. Consider the DTDS $x_{t+1} = 0.2(x_t)^2 + hx_t$ where h is a nonnegative parameter.

(a) [2 points] Find an equilibrium point of the system as a function of h.

$$x^* = 0.2x^{*2} + hx^* \xrightarrow{x^* \neq 0} 1 = 0.2x^* + h \longrightarrow 1-h = 0.2x^*$$

$$\longrightarrow x^* = 5(1-h)$$

Answer: $x^* = \boxed{5(1-h)}$

(b) [1 point] Determine the interval for h where x^* is positive. Remember that $h \geq 0$.

$$5(1-h) \geqslant 0 \longrightarrow h \leqslant 1$$

Answer: $\boxed{0 \leqslant h \leqslant 1}$

(c) [2 points] Define the function $P(h) = hx^*$ where x^* is the equilibrium in part (a). Find h that maximizes $P(h)$ on the interval in part (b).

$$P(h) = 5h(1-h) = 5(h-h^2)$$

$$P'(h) = 5(1-2h) = 0 \longrightarrow h = 0.5$$

h	$P(h)$
0.5	1.25 ← Max
0	0
1	0

Answer: $h = \boxed{0.5}$

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(a) [1 point] The updating function of this DTDS is $f(x) =$ $\boxed{\frac{3+x^2}{1+x}}$

(b) [2 points] The only positive equilibrium point is $x^* =$ $\boxed{3}$

$$x^* = \frac{3+x^{*2}}{1+x^*} \longrightarrow x^* + x^{*2} = 3 + x^{*2} \longrightarrow x^* = 3$$

(c) [2 points] According to the derivative test, is the steady state stable or unstable?

Answer: $\boxed{\text{Stable}}$

$$f'(x) = \frac{2x(1+x) - (3+x^2)}{(1+x)^2}$$

$$f'(3) = \frac{6(4) - (3+9)}{4^2} = \frac{24-12}{16} = \frac{12}{16} \longrightarrow |f'(3)| < 1$$

(d) [1 point] Starting from $x_0 = 2$ calculate x_1, x_2, x_3. Answer: $\boxed{x_1 = 2.33,\ x_2 = 2.53,\ x_3 = 2.66}$

$$x_1 = \frac{3+4}{1+2} = \frac{7}{3} = 2.33 \qquad x_2 = \frac{3+2.33^2}{1+2.33} = 2.53$$

$$x_3 = \frac{3+2.53^2}{1+2.53} = 2.66$$

Question 5. Consider the function $f(x) = xe^{-x}$ on the interval $[0, \infty)$. Follow these steps to graph the function.

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