University of Ottawa
MAT 1332 Practice Midterm Exam
February 2019

These are questions to help you see the scope of the first midterm exam. The format of the midterm will be as follows:

- About 33% multiple-choice and short answer; 67% long answer with full justification required. About 27 points in total; worth 20% of your final grade.
- About 7 questions, some with multiple parts.
- Covering material from : integration: substitution, integration by parts, partial fractions, Riemann sums, improper integrals; differential equations: checking solutions, solving separable differential equations; autonomous differential equations: equilibria, phase line diagrams, stability; complex numbers; matrix algebra.
- Expect to be asked to interpret your mathematical analysis in the context of the application at hand.

Sample instructions:
Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Question B1. [4 points] Calculate

a) $\displaystyle\int_5^{10} \frac{14}{y^2+y-12}\,dy$ b) $\displaystyle\int_1^{e} \frac{1}{x[1+(\ln x)^2]}\,dx$

Question B2. [3 points] Solve the differential equation

$$\frac{dy}{dt} = \frac{7t\cos t}{y}$$

with initial condition $y(0) = 5$.

Question B3. [4 points] Evaluate the integral

$$\int \frac{x^3 + 8x^2 + 5}{x^2 + 5x - 14}\, dx.$$

Question B4. [6 points] For each of the following improper integrals, determine whether it converges, and determine its value if it does.

a) $\displaystyle\int_1^{13} \frac{1}{t \ln t}\, dt$ b) $\displaystyle\int_0^{\infty} \frac{e^{3t}}{1+e^{6t}}\, dt$ c) $\displaystyle\int_1^{\infty} \frac{\ln x}{x^7}\, dx$

Question B5. [4 points] Zombies have invaded campus! Initially, at $t = 0$, there are 5 zombies. They recruit more of the undead to their ghoulish ranks at the rate

$$\frac{dz}{dt} = 8te^{-0.07t},$$

where t is the time in days and z are the number of zombies. How many will be infected if the zombies are recruited forever?

Question B6. (**3 points**) Consider the complex number $z = -1+i$ and the following statements:

(i) $|z| = \sqrt{2}$.
(ii) $z = |z|e^{(\pi i/4)}$.
(iii) $z = |z|\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$.
(iv) $\bar{z} = -1 - i$.
(v) $\dfrac{1}{z} = -\dfrac{1}{2} - \dfrac{i}{2}$.

Which of the following assertions is correct:

A: Only (i) is true. B: (i) and (iii) are true. C: (i), (ii), and (iv) are true.

D: (i), (iv), and (v) are true. E: (ii) and (iii) are true.

Question B7. (**3 points**) Consider the following matrices:

$$X = \begin{bmatrix} 3 & 0 & 3 \\ -1 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \quad , \quad Y = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} 1 & 5 \\ -1 & 0 \\ 3 & 2 \end{bmatrix} .$$

and the following statements:

(i) XY is well defined.
(ii) XZ is well defined.
(iii) $YZ^\top$ is well defined.
(iv) $ZX^\top$ is well defined.
(v) The matrix Y is not invertible.
(vi) $Y + XZ$ is well defined.

Which of the following assertions is correct:

A: (i), (iii), and (vi) are true; B: (iii), (iv), and (v) are true; C : (ii), (iii), and (v) are true;

D: (ii) and (iv) are true; E: (ii) and (vi) are true.

Question B8. (**6 points**) Consider

$$\int_4^{10} \ln(x)\, dx.$$

(a) Compute a left Riemann sum with 6 subintervals to estimate the value of this integral.
(b) Then compute the actual value of this integral.
(c) Explain, using a sketch, why your answers are not equal and which one is expected to be larger.

Question B9. **(5 points)** Consider the initial value problem

$$y' = y^8 x^2, \quad y(1) = 4.$$

(a) Use Euler's method with a step size of $\Delta x = 0.25$ to estimate $y(2)$.
(b) Then solve the differential equation with the given initial condition and give the actual value of $y(2)$.
(c) Suggest a method you could use to obtain a better approximation of $y(2)$ using Euler's method.