University of Ottawa

MAT 1332, Practice midterm (longer than the actual midterm)

February 2019

These are questions to help you see the scope of the first midterm exam. The format of the midterm will be as follows:

- About 33% multiple-choice and short answer; 67% long answer with full justification required. About 27 points in total; worth 20% of your final grade.
- About 7 questions, some with multiple parts.
- Covering material from : integration: substitution, integration by parts, partial fractions, Riemann sums, improper integrals; differential equations: checking solutions, solving separable differential equations; autonomous differential equations: equilibria, phase line diagrams, stability; complex numbers; matrix algebra.
- Expect to be asked to interpret your mathematical analysis in the context of the application at hand.

QUESTION A1. Compute the following integrals:

(a) $\displaystyle\int \frac{x^{3/2} - 5x}{\sqrt{x}} dx$

(b) $\displaystyle\int \left(\sin x + 4x^2 - \frac{6x}{\sqrt[3]{x}}\right) dx$

(c) $\int \sec^2 x \tan x dx$

(d) $\int (x^3 + x)^{10}(3x^2 + 1)dx$

(e) $\int x^4 \ln x dx$

(f) $\int x \sin(x/2) dx$

(g) $\int \frac{e^{1/x}}{5x^2} dx$

QUESTION A2. Use 5 equal subintervals and (a) left endpoints, (b) midpoints, (c) right endpoints to calculate Riemann sums of $f(x) = \sin(x)$ on $[0, \pi/2]$.

QUESTION A3. A population grows according to the differential equation

$$\frac{dN}{dt} = 2\frac{e^{-\sqrt{t}/2}}{\sqrt{t}}$$

.

Determine the net change of this population between $t = 4$ and $t = 16$.

A) $\frac{2}{3}(e^{-2} - e^{-20})$
B) $8(e^{-1} - e^{-2})$
C) $8(e^{-4} - e^{-16})$
D) $6(e^{-1} - e^{-2})$
E) $\frac{5}{3}(1 - e^{-15})$
F) 0

QUESTION A4. Consider the rational function

$$f(x) = \frac{x^2 - 4x}{x^2 - 3x + 2}.$$

(a) Decompose f as a sum of proper fractions.
(b) Use this to determine the indefinite integral $\int f(x)\, dx$.

Question A5. The dynamics of a population of fish in a lake follows a logistic model, to which we have added a negative factor owing to predation; this gives the following model

$$\frac{dN}{dt} = 0.15N(400 - N) - 0.3N$$

where $N(t)$ is the number of fish at time t, and t is measured in weeks.
(a) Find all biologically relevant equilibria.
(b) State and apply the Stability theorem to determine the stability of each of the equilibria.
(c) Draw the phase line diagram for this model.

QUESTION A6. Consider the following integral:

$$\int_0^2 \frac{2x dx}{(x^2-1)^{1/3}}$$

(a) Explain why this integral is improper.
(b) Study its convergence or divergence. If it converges give its value.

QUESTION A7. Solve the following initial value problem:

$$y' = \frac{xy\sin(x)}{y+1}$$

where $y(0) = 1$.

QUESTION A8. Using Euler's method, estimate the value of $y(1)$ if $y(x)$ is the solution of the differential equation

$$y' = xy - x^2$$

satisfying initial condition $y(0) = 1$. Use a step size of $\Delta x = 0.2$.

Question A9. Consider the following vectors and matrices.

$$A = \begin{bmatrix} 1 & -11 & -9 \\ 3 & 0 & 10 \\ 14 & -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -7 \\ 8 & 5 \\ 0 & 1 \end{bmatrix}, \vec{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 5/3 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 1/3 \\ 3/5 \end{bmatrix}.$$

Calculate the following, if possible. If not possible, state why.

a) $A^T\vec{v} + 2B^T\vec{u}$.

b) $\vec{w}\vec{v}^T$

c) $\vec{v}^T\vec{w}$

d) $A\vec{u} + 2\vec{v}^T\vec{w}$

e) AB

f) $B\vec{u}$

g) BA

h) A^2

i) B^2

QUESTION A10. Express the following complex number in polar (Euler) form:

$$z = 1 - 2i.$$

Question A11. Given $u = 3 + 2i$, $v = -1 + 4i$, compute $\overline{u}v + \dfrac{|u|^2}{v}$.

Question A12. For which value(s) of r is $y = x^r \ln(x)$ a solution of the differential equation

$$y' = \frac{y}{x}\left(\frac{3\ln(x) + 1}{\ln(x)}\right)?$$