Lecture 4

January 17, 2018 8:27 AM

6.3 - Volume by sylindrical shells

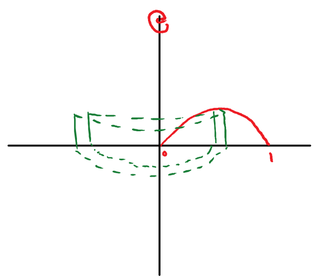
$$V = \int_a^b 2\pi x\, f(x)\, dx$$

Example 9:

$y = x - x^2$

$y = 0$

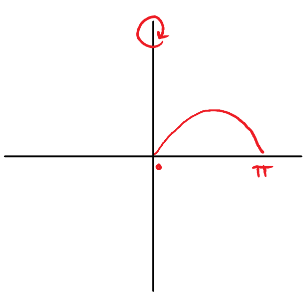
$$x - x^2 = 0$$
$$x(1-x) = 0$$
$$x = 0, 1$$



$$V = \int_0^1 2\pi x(x - x^2)\, dx$$
$$= 2\pi \int_0^1 (x^2 - x^3)\, dx$$
$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right] \Big|_0^1$$
$$= \boxed{\frac{\pi}{6}}$$

Example 10

$f(x) = \sin(x) \qquad 0 \leq x \leq \pi$



$$V = \int_0^\pi 2\pi x (\sin(x))\, dx$$
$$= 2\pi \int_0^\pi x \sin(x)\, dx \qquad \begin{array}{ll} f = x & g' = \sin(x) \\ f' = 1 & g = -\cos(x) \end{array}$$
$$= 2\pi \left[-x\cos(x) \Big|_0^\pi - \int_0^\pi -\cos(x)\, dx \right]$$
$$= 2\pi \left[\pi + \sin(x) \Big|_0^\pi \right]$$
$$\boxed{= 2\pi^2}$$

Work: → if force is constant

* if force is not constant

Work: → if force is constant

→ $W = F \cdot d$

$= N \cdot m$ or $=$ Pound · Feet

$=$ Joules(J) $=$ foot·pound (ft - lb)

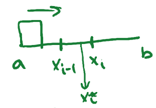
$= 1.36$ J

$F = m \cdot g$

$m = v \cdot d$

$=$ volume · density

* if force is not constant



$\Delta x = \frac{b-a}{n}$

x_i^* is a sample point on $[x_{i-1}, x_i]$

→ The force is approximately $F(x_i^*)$

$W_i = F(x_i^*)\Delta x$

$W = \sum_{i=1}^{n} F(x_i^*)\Delta x$

$W = \lim_{n \to \infty} \sum_{i=1}^{n} F(x_i^*) dx$ → $W = \int_a^b F(x)\,dx$

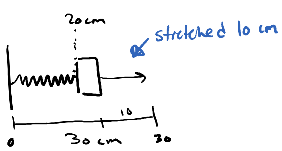
$$\boxed{W = \int_a^b F(x)\,dx}$$

Example 11: Work needed to stretch a spring

Hooks law: Hooks law states that the force required to maintain a spring stretched x units beyond its natural length.

$\boxed{f(x) = kx}$ → distance to neutral point → spring constant

Example 12:



$f(x) = kx$

$25 = k(0.1)$

$k = 250$

20 | 25 $\quad x = 5cm = 0.05m$

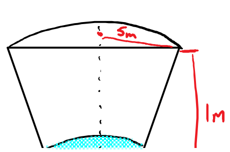
$$W = \int_0^{0.05} 250x\,dx$$

$$= 125x^2 \Big|_0^{0.05}$$

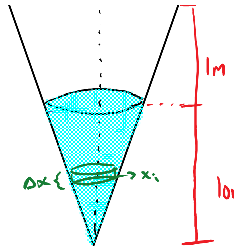
$$= 125(0.05)^2 - 125(0)^2$$

$$= 0.3125 \text{ J}$$

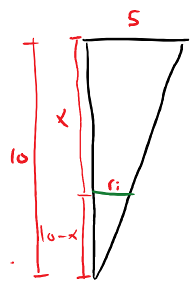
Example 13:



→ The water extends from a depth of 1m to 10m. lets divide [1, 10] into n subintervals and choose x_i in the sub interval



Sub interval



$$\frac{r_i}{5} = \frac{10-x}{10}$$

$$r_i = \frac{1}{2}(10-x)$$

→ So an approximation for $v_i = \pi r^2 \Delta x = \pi \frac{1}{4}(10-x)^2 \Delta x$

$$m_i = v \cdot d$$
$$= \frac{\pi}{4}(10-x)^2 \Delta x \cdot 10^3$$
$$= 250\pi(10-x)^2 \Delta x$$

→ The Force that is required to raise this layer, must overcome the force of gravity

$$F_i = m_i \cdot g$$
$$= (250\pi(10-x)^2 \Delta x)(9.8)$$
$$= 2450\pi(10-x)^2 \Delta x$$

→ Each particle in the layer travels a distance x_i →

$$W_i = F_i \cdot x_i = 2450\pi(10-x)^2 x_i \cdot \Delta x$$

$$\Rightarrow W = \lim_{n\to\infty} \sum_{i=1}^{n} 2450\pi(10-x)^2 x_i \cdot \Delta x$$

$$= \int_1^{10} 2450\pi(10-x^2)x\,dx$$

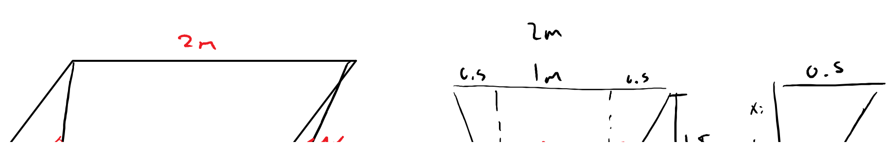
$$= 2450\pi \int_1^{10} (100-20x+x^2)x\,dx$$

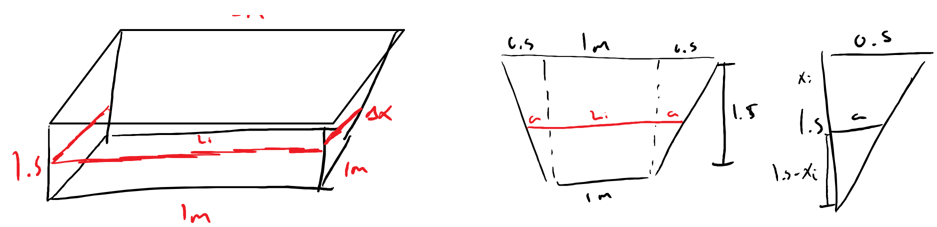
$$= 2450\pi\left[50x^2 - \frac{20x^3}{3} + \frac{x^4}{4}\right]\Bigg|_1^{10}$$

$$= 1934888\pi$$

$$\boxed{= 6.079 \times 10^6 \text{ J}}$$

Example 14:





$$L_i = 1 + 2a$$

$$\frac{a}{1/2} = \frac{3/2 - x_i}{3/2}$$

$$3a = \frac{3}{2} - x_i$$

$$a = \frac{1}{2} - \frac{x_i}{3}$$

$$L_i = 1 + 2\left(\frac{1}{2} - \frac{x_i}{3}\right)$$

$$= 2 - \frac{2x_i}{3}$$

Volume of the cross-sectional

$$V_i = L_i \times 1_m \times \Delta x$$

$$= \left(2 - \frac{2x_i}{3}\right)\Delta x$$

$$m_i = v \cdot d$$

$$= \left(2 - \frac{2x_i}{3}\right)\Delta x \cdot 10^3$$

$$F_i = m_i \cdot g$$

$$= \left(2 - \frac{2x_i}{3}\right)\Delta x \cdot 10^3 \cdot 9.8$$

$$W_i = F_i \cdot d$$

$$= \left(2 - \frac{2x_i}{3}\right)\Delta x \cdot 10^3 \cdot 9.8 \cdot x_i$$

$$W = \lim_{x \to \infty} \sum_{i=1}^{n} F_i x_i$$

$$= \int_0^{1.5} 9800\left(2 - \frac{2x_i}{3}\right) x \, dx$$

$$= 9800\left[x^2 - \right.$$

Volume, mass, Force then work