Lecture 2 → Lecture 1 continued

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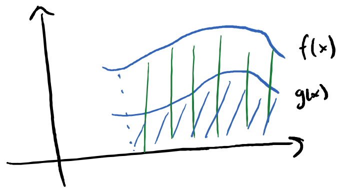
# The comparison Test

→ if $0 \le g(x) \le f(x) \Rightarrow 0 \le \int_a^\infty g(x)\,dx \le \int_a^\infty f(x)\,dx$




* if $\int_a^\infty f(x)\,dx$ converges $\Rightarrow \int_a^\infty g(x)\,dx$ must also converges

* if $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)\,dx$ must also diverges

# Useful Integrals for comparison

- [P-Test] $\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1} & \text{if } p>1 \\ \infty & \text{if } p\le 1 \end{cases}$ * see example 4 of textbook pg 530
- [P-Test] $\int_0^1 \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{1-p} & \text{if } p<1 \\ \infty & \text{if } p\ge 1 \end{cases}$
- $\int_0^\infty e^{-ax}\,dx = \begin{cases} \frac{1}{a} & \text{if } a>0 \\ \infty & \text{if } a\le 0 \end{cases}$

## Ex 6

a) $$\int_2^\infty \frac{1}{x^4+6x+5}\,dx$$

$x \to \infty$ $\frac{1}{x^4+6x+5}$ behaves like $\frac{1}{x^4}$

$$x^4 < x^4+6x+5 \Rightarrow \underbrace{\frac{1}{x^4+6x+5}}_{g(x)} < \underbrace{\frac{1}{x^4}}_{f(x)}$$

The smaller function converges, so the larger one converges as well

$\int_2^\infty \frac{1}{x^4}\,dx$ → converges by the P-Test $(p=4)$

$\Rightarrow \int_2^\infty \frac{1}{x^4+6x+5}\,dx$ must converge as well

b) $\displaystyle\int_2^{\infty} \frac{2-\cos(x)}{x}\,dx$ → Find a boundary, then use comparison test

$-1 \le \cos x \le 1 \qquad 1 \le 2-\cos x \le 3$

$x \to \infty$
$x > 0$
$\qquad \frac{1}{x} \le \frac{2-\cos x}{x} \le \frac{3}{x}$

$\displaystyle\int_2^{\infty} \underbrace{\frac{1}{x}\,dx}$ → diverges by the P-Test

because the small function diverges the larger function diverges too

$\Rightarrow \displaystyle\int_2^{\infty} \frac{2-\cos(x)}{x}\,dx$ must also diverge as well

c) $\displaystyle\int_0^{\infty} e^{-x^2}\,dx = \int_0^{1} e^{-x^2}\,dx + \int_1^{\infty} e^{-x^2}\,dx$

$\displaystyle\int_1^{\infty} e^{-x^2}\,dx$

$x \to \infty \quad x \ge 1 \quad x^2 \ge x \Rightarrow -x^2 \le -x \Rightarrow \underbrace{e^{-x^2}}_{g(x)} \le \underbrace{e^{-x}}_{f(x)}$

$$\int_1^{\infty} e^{-x}\,dx = \lim_{d\to\infty} \int_1^{d} e^{-x}\,dx$$

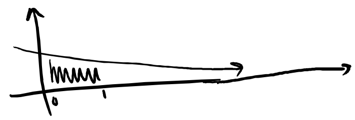
$$= \lim_{d\to\infty} \left(-e^{-x}\right)\Big|_1^d$$

$$= \lim_{d\to\infty} \left[-\cancel{e^{-d}}_{0} + e^{-1}\right]$$

$$= \boxed{\frac{1}{e}}$$

$\Rightarrow \displaystyle\int_1^{\infty} e^{-x}\,dx$ converges $\Rightarrow \displaystyle\int_1^{\infty} e^{-x^2}\,dx$ must also converges

$\int_0^1 e^{-x^2}\,dx \Rightarrow$ Just an ordinary definite Integral $\Rightarrow$ convergent




$\Rightarrow \int_0^\infty e^{-x^2}\,dx$ Converges

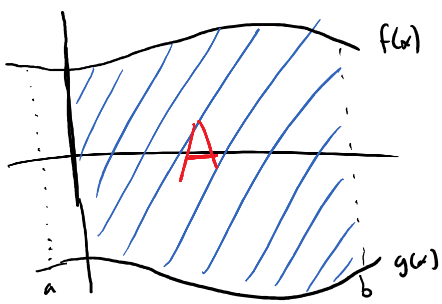
## Start of Lecture 2

# Chapter 6 Applications of Integrals

## 6.1 Area between two curves:

→ $f(x)$ and $g(x)$ are continuous on $[a,b]$

→ $f(x) \geq g(x)$ $\quad x \in [a,b]$

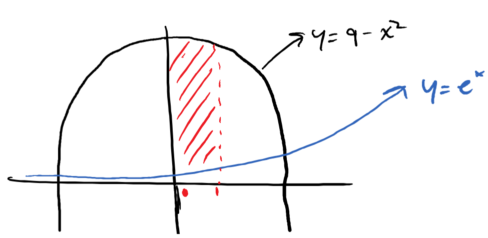



$$A = \int_a^b \left(f(x) - g(x)\right) dx$$

**Ex 1** Find area between Functions

$f(x) = 9 - x^2$ $\quad x = 0$ and $x = 1$

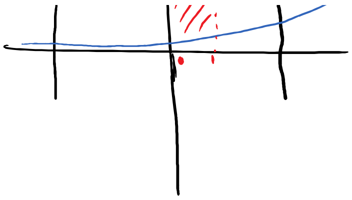
$g(x) = e^x$




$$A = \int_0^1 \left(9 - x^2 - e^x\right) dx$$

$$= \left(9x - \frac{x^3}{3} - e^x\right)\Big|_0^1$$

$$= \frac{29}{3} - e$$



$$= \frac{29}{3} - e$$

$$\boxed{= 6.95}$$

## Ex 2

$f(x) = 2 - x^2$

$g(x) = x^2$

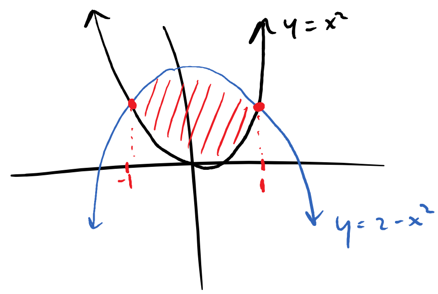
* when it doesn't tell you boundaries, you need to find point of intersection first

POI

$2 - x^2 = x^2$

$2x^2 = 2$

$x = \pm 1$




$$A = \int_{-1}^{1} \left(2 - x^2 - x^2\right) dx$$

$$= \int_{-1}^{1} \left(2 - 2x^2\right) dx$$

$$= \left(2x - \frac{2}{3}x^3\right)\Big|_{-1}^{1}$$

$$\boxed{= \frac{8}{3}}$$

## Ex 3

$y^2 = x + 2$ and $y = x$

Change it → $x = y$

$x = y^2 - 2$

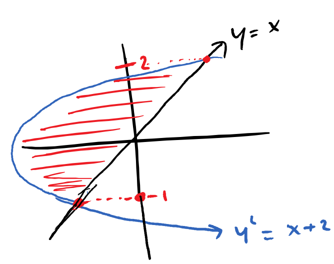
Find point of intersection

$y = y^2 - 2$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$\boxed{\begin{array}{l} y = 1 \\ y = 2 \end{array}}$




You get a negative and you can't have that so you have to rewrite to get positive

$y = 1$
$y = 2$


You get a neg... so you have to rew...

$$A = \int_{-1}^{2} (y^2 - 2 - y)\,dy \quad\longrightarrow\quad A = \int_{-1}^{2} \left(y - (y^2 - 2)\right) dy$$

$$= \left(\frac{y^3}{3} - 2y - \frac{y^2}{2}\right)\Big|_{-1}^{2} \qquad = \left(\frac{y^2}{2} - \frac{y^3}{3}\right.$$

$$\boxed{= -\tfrac{9}{2}} \qquad = 9/2$$

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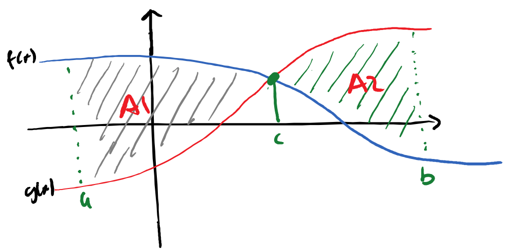
## Area Between Intersecting curve:




$A = A_1 + A_2$

$A = A_1 + A_2$

$$A = \int_a^c [f(x) - g(x)]\,dx + \int_c^b [g(x) - f(x)]\,dx$$

Ex 4

$y = x$
$y = x^3$

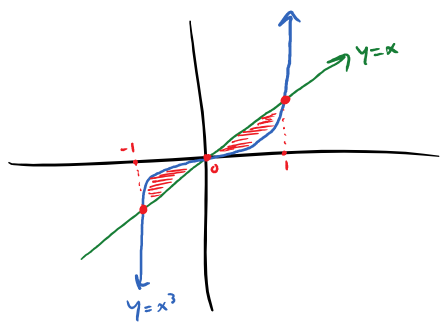
$x = x^3$

$0 = x^3 - x$

$0 = x(x^2 - 1)$

$x = 0$

$x = \pm 1$ 3 Points




$$A = A_1 + A_2$$

$$= \int_{-1}^{0} (x^3 - x)\,dx + \int_0^1 (x - x^3)\,dx$$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1$$

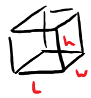
$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4}$$

$$\boxed{= \frac{1}{2}}$$

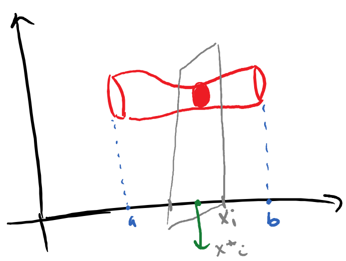
# Section 6.2: volume

→ Rectangular Box → $V = \underbrace{L \times h}_{A} \times w$




→ Cylinder → $V = \underbrace{\pi r^2}_{A} h$


→ $V = A \times h$




→ Chop the interval $[a,b]$ into $n$ subintervals

$\Delta x = \frac{b-a}{n}$ $\quad A(x^*i)$

$$V_i \simeq A(x^*i) \underbrace{\Delta x}_{h}$$

$$V \simeq \sum_{i=1}^{n} A(x_i^*) \Delta x$$

$n \to \infty$ and $\Delta x \to 0$

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i) \Delta x$$

$$= \int_a^b A(x)\, dx$$