Radioactive decay

As we discussed earlier excited nuclei will decay and eventually return to stability. So, radioactive decay is the process by which an unstable atomic nucleus loses energy (in terms of its rest mass) by emitting radiation, such as an alpha particle, beta particle with neutrino, or a gamma ray. A material containing such unstable nuclei is considered radioactive. Below is a brief discerption of the most likely types of radioactive decay:

- decay to a lower state by emitting a photon with an energy equal to the difference between the two energy levels. They are called gamma (γ) rays.

- loose its energy through internal conversion. In This process excitation energy is transformed into the inner most electron. The electron is then ejected with energy equal to the nuclear transition minus the ionization energy.

- undergo β decay. If the nucleus is lacking enough neutrons necessary for stability, then the nucleus undergoes β^{+} $(positron)$ decay. For example

  $$ {}^{15}_{8}O \xrightarrow{\beta^{+}} {}^{15}_{7}N + \nu $$

  If the nucleus has more neutrons than needed for stability, it will, most likely, undergo β^{-} decay. For example

  $$ {}^{19}_{8}O \xrightarrow{\beta^{-}} {}^{19}_{9}F + \bar{\nu} $$

  $\bar{\nu}$ is antineutrino,

  If the daughter nucleus (F in this case) is also unstable, this leads to a decay chain, as follows

  $$ {}^{20}_{8}O \xrightarrow{\beta^{-}} {}^{20}_{9}F \xrightarrow{\beta^{-}} {}^{20}_{10}Ne\ (stable) $$

- emit α particles. This type of decay occurs mostly for heavy nuclei (Z>80). The emission of α particles reduces the atomic number by 2 and the mass number by 4

  $$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \alpha({}^{4}_{2}He) $$

- undergo an electron capture . In this case, if the nucleus is lacking neutron, an atomic electron interact with one of the protons and a neutron is formed during the union. This leaves vacancy in electron cloud which is later filled by another electron and leads to γ ray emission.

Radioactivity decay calculations

I am sure that you have seen a similar example in your probability and statistics course, if you have taken one:

If we pick a light bulb from a batch of a large number of these bulbs comes out of the factory, it is impossible to say with certainty that this particular light bulb will operate, say, for a 1000 hours or 2000 hours before it burns. However, from the experience, using thousands and thousands of this item over many years, we can say that the average life of the light bulb is e.g. 1500 hours, or the probability of its failure in the first 100 hours is e.g. 0.02.

This example is a general representation of the concept of statistically large population where some random event can occur to any member of that population. Such a process is called a stochastic process, and we deal with it in a probabilistic way.

Similarly, we know that a collection of unstable nuclei will decay throughout the time, and eventually to a stable state. But there is no way we can predict with certainty if a particular nucleus decays in a specific period of time. And by the way, it is not useful to know that. Therefor the radioactive decay is a random stochastic process with no preferred direction (isotropic). It can only be predicted through averaging and statistical treatment. What we can obtain from experiments is the probability per unit time of the decay of a nucleus, call it λ. The fundamental law governing decay processes says **that λ is a constant for a particular decay process. λ is also called the decay constant.**

If we have a collection of N nuclei of a radioactive isotope at time t (N is a very large number). Let Δt be a time period during which ΔN nuclei experienced radioactive decay, then

$$\lambda = \frac{\Delta N / N}{\Delta t} \qquad (1)$$

In this course we will consider calculations of three scenarios which are widely common in nuclear applications. We will start with the simplest case and add a couple of intricacies later.

Scenario 1

Let us assume that at $t=0$, we are tracking a collection of N_o of radioactive, and we want to predict how $N(t)$ is changing with time, given that λ is constant. The easy way to solve this problem is to consider equation 1 in the limit of $\Delta t \to 0$ then, by definition, the Δ operator becomes a total derivative.

$$\lambda = \lim_{\Delta t \to 0} \frac{\Delta N / N(t)}{\Delta t} = \frac{1}{N(t)} \frac{dN(t)}{dt} \qquad (2)$$

Since *N(t)* is a decreasing function of t and the decay constant is considered always to be positive, the *N(t)* should be a negative. With this understanding, rearranging equation 2 to be

$$\frac{dN(t)}{dt} + \lambda N(t) = 0 \qquad (3)$$

Equation 3 is a first order differential equation subject to the initial conditions $N(0)=N_o$. the solution of equation 3 is given by

$$N(t) = N_o e^{-\lambda t} \qquad (4)$$

A more convenient format of writing equation 4 is obtained by multiplying the equation by λ, where λN is defined as the activity α,

$$\alpha(t) = \alpha_o e^{-\lambda t} \qquad (5)$$

The time during which the activity falls to half its value is called the half-life, denoted by $T_{1/2}$. which can be shown to be:

$$T_{1/2} = \frac{0.693}{\lambda} \qquad (6)$$

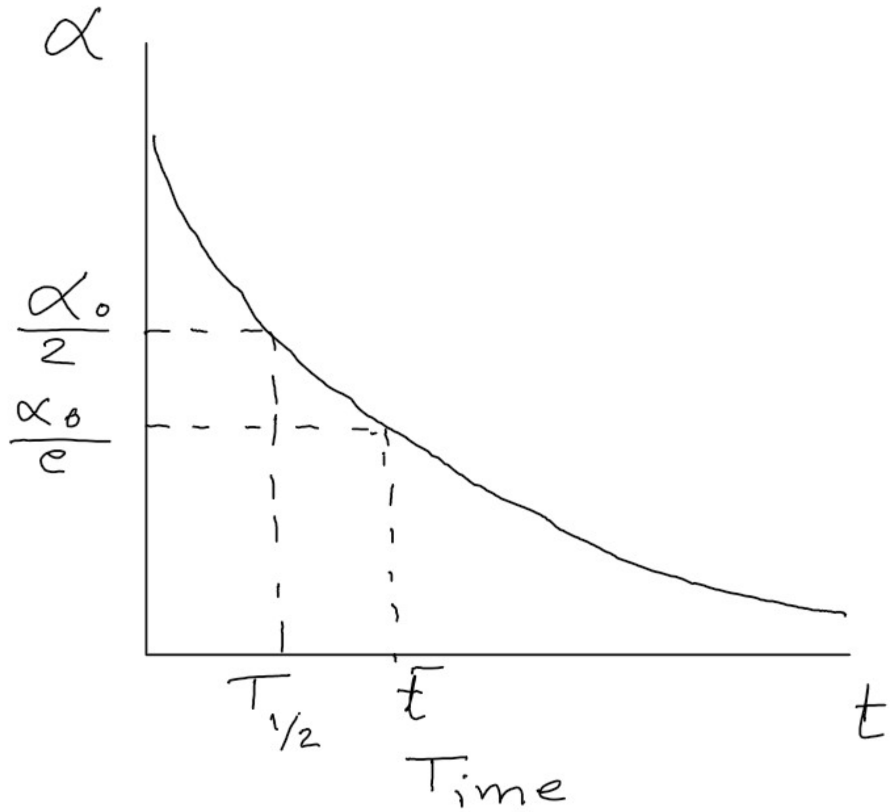
Since half-lives are widely reported, tabulated and known parameter, the decay equation 5 can be written in terms of the half-life as

$$\alpha(t) = \alpha_o e^{-\frac{0.693t}{T_{1/2}}} \qquad (7)$$

We can also show that the average life expectancy (the mean life of a radioactive nucleus $\bar{t}$). It is given by $\bar{t} = 1/\lambda$. The activity falls by 1/*e* of its initial value after $\bar{t}$. Also

$$\bar{t} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44T_{1/2} \qquad (8)$$

The following figure shows the exponential decay of a typical radioactive sample.



Scenario #2

Frequently we are faced with the problem of concurrent radioactive nuclei production and decay. Such situation can be experienced in nuclear reactors or in target chamber of an accelerator.
let us assume, it is produced at constant rate R atoms per unit time. The balance equation in this case will be

$$\frac{dN(t)}{dt} + \lambda N(t) = R \qquad (8)$$

The solution of equation 8 is

$$N(t) = N_o e^{-\lambda} + \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right) \qquad (9)$$

In terms of activity

$$\alpha(t) = \alpha_o e^{-\lambda t} + R\left(1 - e^{-\lambda t}\right) \qquad (10)$$

If α_o=0, the asymptotic activity is R, and the asymptotic number of nuclei is R/λ

Scenario #3

When a radionuclide decays, it does not disappear, but is transformed into a new nuclear species of lower energy and often differing *Z* and *A*. The equations of radioactive decay discussed so far have focused on the decrease of the parent radionuclides but have ignored the formation (and possible decay) of daughter, granddaughter, etc., species. It is the formation and decay of these "children" that is the focus of this section. Let us consider the following decay chain

$$A \xrightarrow{radioactive\ decay} B \xrightarrow{radioactive\ decay} C$$

Where A in this chain decay reaction is the parent radionuclide, B is the daughter, and C is the granddaughter. The rate of change of the number of the daughter radionuclides B at any time t is governed by the production of B (decay of A) less the decay of B. Putting this into equation format we get

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \qquad (11)$$

Substitute for N_A from equation 4, we get

$$\frac{dN_B}{dt} = \lambda_A N_{Ao} e^{-\lambda_A t} - \lambda_B N_B \qquad (12)$$

Equation 12 is a first order differential equation which can be solved by multiplying both sides of the integrating factor ($e^{-\lambda_B t}$). Given that at *t=0,* $N_A=N_{Ao}$ and $N_B=N_{Bo}$, The solution of equation 12 will be

$$N_B(t) = N_{Bo} e^{-\lambda_B t} + \frac{N_{Ao}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) \qquad (13)$$

Multiplying equation 13 by λ_B, it can be written in terms of activity as:

$$\alpha_B(t) = \alpha_{Bo} e^{-\lambda_B t} + \frac{\alpha_{Ao}\lambda_B}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) \qquad (14)$$

Some Comments

- N_o is a collection of atoms and can be expressed in grams, moles, or number of atoms.

- The half life parameter occupies a very wide span of times, from seconds to millions of years.
- The SI units for the activity α is the Becquerel (Bq). It is a derived unit and defined as the activity of a quantity of radioactive material in which one nucleus decay per second.
- Another widely used, non- SI unit is the Curie (Ci) defined as the mass of radium (^{226}Ra) emanation in equilibrium with one gram of radium.

$$1\ Ci = 3.7 \times 10^{10} Bq = 37\ GBq$$
$$1\ Bq = 2.703 \times 10^{-11} Ci = 27PCi$$

Neutron interaction with matter

Since the neutrons have no electrical charges, they can travel through the electron cloud of the atom and through electrical field of the nucleus unaffected by the charges of the protons and electrons. That is the reason neutron interactions with matter, particularly with heavy metals, play a vital role in the design and operation of nuclear reactors. Neutron interaction with matters is a type of nuclear reaction and must satisfy its conservation laws. Because neutrons are neutral and not affected by the electron clouds around the nucleus they directly interact with the nuclei. In the following we will discuss the different possible outcomes resulting from the interaction of neutron with the nucleus.

As the neutrons diffuse through the matter, they may enter into a number of reactions with the nuclei of various isotopes. These reactions fall into two main categories:

1. Scattering
2. Absorption

Each category is further divided into different subcategories. Each reaction can be thought of as to take place in two steps

Step 1: Nucleus absorbs the neutron to form a compound nucleus at excited states. The compound nucleus has also the added binding energy of the last (absorbed) neutron to the original binding energy. The excitation energy comes from the kinetic energy of the absorbed neutrons.

Step 2 will depend on the energy state of the compound nucleus. Here are the different possible outcomes.

- In scattering reactions, the compound nucleus rapidly expels a neutron with lower kinetic energy. The excess energy remains in the nucleus. if this energy is in the form of internal energy leading to excited nucleus, then the reaction is called **inelastic scattering**. It generally takes place between high energy neutron and heavy nucleus the excess energy is released as a gamma ray (γ) and it is denoted by (n, γn). If the extra energy in the compound nucleus is solely kinetics then a form of **elastic scattering** has occurred, In this type of reactions, light nuclei are the most effective for slowing down neutrons. A neutron colliding with a heavy nucleus rebounds with little loss of speed and transfer little energy, denoted by (n, n)

- Charged particle reaction (falls under absorption type reaction): In this type of reactions the excited compound nucleus de-energizes by emitting a changed particle, either a proton or α particle producing a nucleus of different isotope (that is why it is also called a transmutation reaction). For example

$$ {}_{0}^{1}n + {}_{8}^{16}O \rightarrow {}_{7}^{16}N + {}_{1}^{1}p $$

${}_{7}^{16}N$ is radioactive with a half-life of 7.1 seconds. It is a beta emitter. It also emits very penetrating gamma ray. The reaction is denoted by (n, α) or (n,p).

- Radiative capture (also falls under absorption type reaction): The most Common nuclear reaction. The compound nucleus emits only a gamma photon. The product nucleus is an isotope of the same element. The simplest radiative capture example when hydrogen nucleus absorbs neutron to produce deuterium,

$$ {}_{1}^{1}H + {}_{0}^{1}n \rightarrow {}_{1}^{2}H + \gamma_{ray} $$

The deuterium itself undergoes radiative capture reaction to form tritium $({}_{1}^{3}H)$. The tritium is unstable and represents a major radiation hazard in CANDU reactors

- Fission reaction (also falls under absorption type reaction) when a heavy compound nucleus have sufficient energy to split apart, it is said to have undergone fission. This reaction is the principal source of nuclear energy.

The concept of neutron cross-section

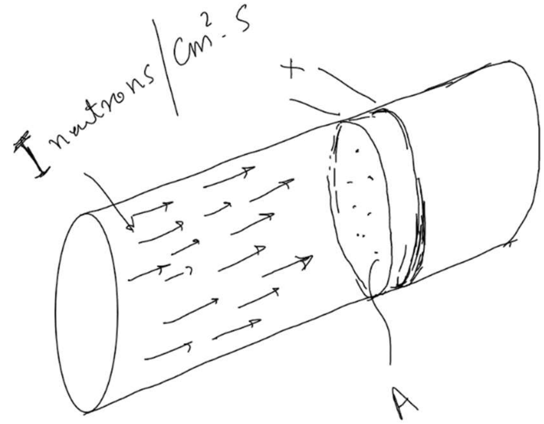
When a stream of neutrons of intensity I neutrons/cm^2.s is fired on a thin target sample shown below, a small fraction of neutrons will collide with a nucleus. The rest will penetrate through the target untouched. The number of collisions is found to be proportional to the neutron beam intensity, the area, the thickness, and the atomic number density of the target. Assuming that all neutrons in the beam have the same speed (monoenergetic), then the number of collisions per second can be expressed as

$$ number\ of\ collision\ per\ second = \sigma IAXN \qquad (15) $$

Where σ is the constant of proportionality has a dimension of L^2. and represents the likelihood (probability) of a particular reaction. The last three factors in equation 15 (*AXN*) represents the number of nuclei in the target. Therefore, the number of collisions per second for a specific target nucleus is ($I\,\sigma$). As such, σ represents the likelihood (probability) of a particular reaction, and since it has the dimension of L^2 , it called the nucleus cross-section, and measured by a unit called barn.

$$1\ barn = 10^{-24} cm^2$$

Different nuclei have different probabilities of reacting with a neutron. A given target nucleus, struck by a neutron, has a different likelihood of undergoing any of the above reactions. Neutron cross-section represents the probability that a reaction occurs when neutrons bombard a target nucleus. A nucleus has different cross sections for different reactions; subscripts denote the type of cross-section. σ_a is the absorption cross-section, σ_s is the scattering cross-section, σ_f is the fission cross-section and so on.



The total cross section σ_t measures the likelihood that an interaction of any type will take. As such, it is measured by the sum of all possible reactions

$$\sigma_t = \sigma_s + \sigma_a \qquad (16)$$

Similarly $\sigma_a = \sigma_f + \sigma_\alpha + \sigma_p + \sigma_\gamma + \cdots \qquad and \qquad \sigma_s = \sigma_e + \sigma_i \qquad (17)$

Based on the above definitions,

$$the\ total\ number\ of\ collisions\ in\ the\ entire\ target\ per\ second = I\sigma_t NAX$$

The number of collisions per unit volume of the target can be obtained by dividing the last equation by the volume of the target (*AX*),

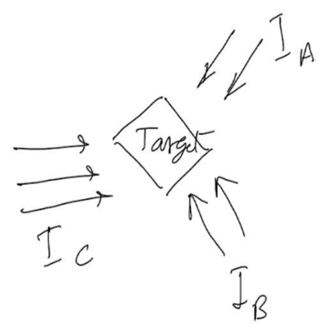
$$the\ total\ number\ of\ collisions\ in\ the\ entire\ target\ per\ second\ per\ cm^3 = I\sigma_t N$$

The las two factors ($\sigma_t N$) is defined as the macroscopic cross-section and denoted by Σ_t . It has a dimension of L^{-1}. The additive rules of equations 16 and 17 are equally applicable to Σ.

Neutron Flux

In the previous section, we used the neutron beam intensity to express a unidirectional stream of monoenergetic neutrons. When a number of neutron streams traveling from different directions toward a given target as shown below, the total number of neutrons per unit volume of each beam n_A, n_B, n_C times the velocity is called the neutron flux ϕ.

$$\phi = (n_A + n_B + n_C + \cdots)\text{v}$$



It is obvious that the number number collisions per unit volume per unit time (also called the rate of reaction *F*) is given by

$$F = \Sigma_t \phi \qquad (18)$$

Neutron attenuation

Suppose that a neutron beam of intensity I_o is directed toward a thick target (Thickness X). A neutron detector is located at a distance behind the target. The detector will only Counts
Those neutrons that are not involved in collision as they travel through the target. Let *I(x)* be the intensity of the neutrons that have **not** collided after traveling a distance x through the target. Then after travelling a distance *dx* through the target, the intensity of the neutron beam will be decreased by *dI,* then

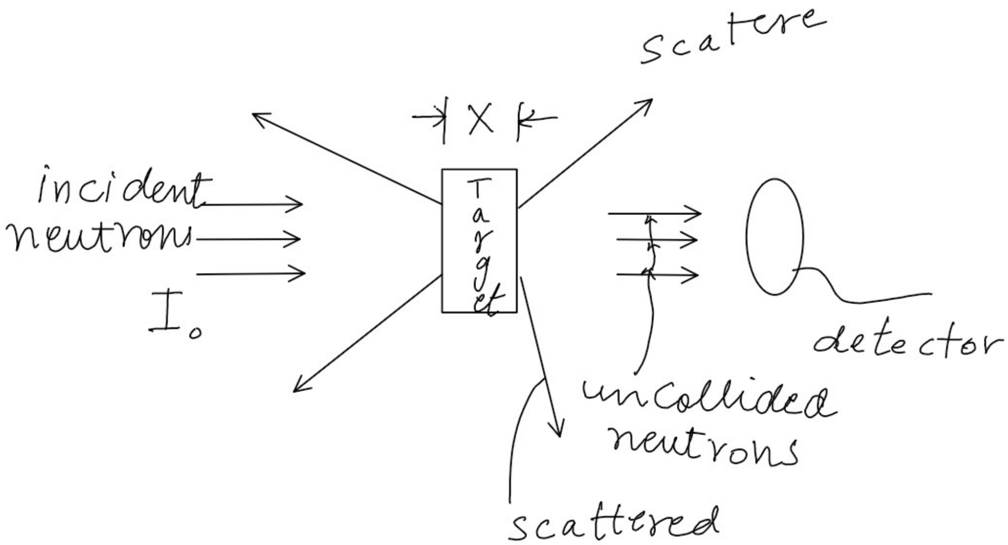
$$dI = -\Sigma_t I(x) dx$$

Or

$$\frac{dI(x)}{dx} = -\Sigma_t I(x) \qquad (19)$$

Equation 19 is a linear first order differential equation subject to the boundary condition $I(0)=I_o$
It is very similar to equation 3 above. The solution of equation 19 is given by

$$I(x) = I_o e^{-\Sigma_t x} \qquad (20)$$



Thus the intensity of the incident beam decreases exponentially as it traverses through the target.

let *f(x)dx* be the probability that a neutron has traversed through the target a distance *x* and will involve in a collision within the neighbouring *dx*, then

$$f(x)dx = probability\ of\ neutron\ survival\ until\ x \times probability\ of\ collision\ in\ dx$$

That is

$$f(x)dx = \frac{I(x)}{I_o} \times \frac{-dI}{I(x)}$$

Substitute for $I(x)/I_o$ from equation 20 and substitute for $dI/I(x)$ from equation 19 to obtain

$$f(x) = \Sigma_t e^{-\Sigma_t x} \qquad (21)$$

f(x) is the probability distribution function (also called probability density function or pdf) of neutron collision.

The average of *f(x)* represents the average distance travelled by a neutron before colliding with a nucleus. It is defined as the mean free path λ. It is one of the main determining factors of the size of a nuclear reactor.

$$\lambda = \int_0^\infty x f(x) dx = \int_0^\infty x \Sigma_t e^{-\Sigma_t x} dx = \frac{1}{\Sigma_t} \qquad (22)$$

Examples

1. (a)Radioactive substance has activity of 6144 Bq. How many half-lives will it lake to fall to 6 Bq.
   (b) what will-be the activity 6 half-lives later, if the initial activity is 192 Bq.

2. The radioisotope sodium-24, half-life 15 h, is used to measure the flow rate of salt water by irradiation of stable $^{23}_{11}Na$ with neutrons, suppose that we produce 5 micrograms of the isotope. How much do we have at the end of 24 h?

3. How long after a sample is placed in a reactor is it before the sample activity reaches 75% of the maximum activity? Assume the production of a single reactive species at a constant rate R sec^{-1}, the decay rate λ, and there is no initial activity in the sample.

4. Radon-222 is seeping into a basement of a house at a rate of $6.6x10^{10}$ atoms/sec. Radon is a gas and it has a half-life of 3.8 days and is one of the members of the U-238 decay chain. The basement has a volume of 230 m^3. Supposing there is no ventilation at all, what will be the asymptotic concentration of Rn-22.

5. Consider the decay chain $A \rightarrow B \rightarrow C \rightarrow$ with no atoms of B present at t=0

   a. Show that the activity of B rises to a maximum value at time t_m given by
   $t_m = \frac{1}{\lambda_B - \lambda_A} ln\left(\frac{\lambda_B}{\lambda_A}\right)$ at which the activities of A and B are equal

   b. Show that, for $t < t_m$, the activity of B is less than that of A, whereas the reverse is the case for $t > t_m$.

6. Name the following nuclear reactions:

$^{2}_{1}H(n,n)^{2}_{1}H,\quad ^{238}_{92}U(n,\gamma n)^{238}_{92}U,\ ^{16}_{8}O(n,p)^{16}_{7}N,\ ^{10}_{5}B(n,\alpha)^{7}_{3}Li,\ ^{59}_{27}Co(n,\gamma)^{260}_{27}Co,\ ^{1}_{1}H(n,\gamma)^{2}_{1}H,$

7. A beam of 1 MeV neutrons of intensity 5 x10^8 neutrons/cm^2-sec strikes a thin ^{12}C target. The area of the target is 0.5 cm^2 and is 0.05 cm thick. The beam has a crosssectional area of 0.1 cm^2. At 1 MeV, the total cross-section of ^{12}C is 2.6 b. (a) At what rate do interactions take place in the target? (b) What is the probability that a neutron in the beam will have a collision in the target?

8. Referring to Example 3. 1 , calculate the (a) macroscopic total cross-section of C-12 at 1MeV; (b) collision density in the target.

9. There are only two absorption reactions-namely, radiative capture and fission-that can occur when 0.0253-eV neutrons interact with U-235. The cross-sections for these reactions are 99 b and 582 b, respectively. When a 0.0253-e V neutron is absorbed by 235U, what is the relative probability that fission will occur?