Some physics concepts

Before delving into the sources and production of the nuclear energy we need introduce the theories and their results that made the utilization of nuclear energy possible. The three major advancements relevant to the production to the nuclear energy are:

1. The equivalence of mass and energy
2. The wave-particle duality
3. Quantum Mechanics

In 1905 Einstein showed that the mass of an object *m* is not constant (as thought by Newton), but increases with its speed v according to

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

where m_o is the object's "rest mass", c is the speed of lights $3\text{x}10^8$m/s.

From practical engineering view point, relativistic effects in macroscopic world can be safely neglected without incurring any noticeable errors. Yet, they can be significant at atomic and subatomic scales.
The relative mass equation above shows that an object with rest mass m_o cannot travel at the speed of light, as the mass will increase with out limit and the object will obtain infinite kinetic energy. Other results of the theory of relativity include the relativistic length in the direction of motion

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}} \qquad (2)$$

and the dilatation of time is

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3)$$

The most renowned result of the special relativity is the equivalence of mass and energy.

$$E = mc^2 \qquad (4)$$

This result, which can be proved mathematically, shows that mass and energy can be converted from one to the other. Fundamentally, production of nuclear energy is a manifestation of this equation, where a tiny bit of the nucleus mass is converted into energy. In this course, we are trying to understand the conditions under which such process is made possible. We will also try to understand how to control the conversion process so that we can harness and safely benefit from nuclear energy.

E in equation (4) represents the total energy of a body of the relative mass *m* given by equation (1). Based on that, we can say that the kinetic energy *(K.E.)* is the difference between the total energy *E* and the energy of the rest mass m_o.

$$K.E. = mc^2 - m_o c^2 = m_o c^2 \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] \qquad (5)$$

At macroscopic scales, the speed of common objects (cars, airplanes, rockets, bullets, …etc.) is much less than that of the light (v<< c). Therefore, If the term $\left(1 - {v^2}/{c^2}\right)^{-1/2}$ in equation 5 is expanded using Maclaurin series and retaining only the first two terms of the series, we can retrieve the classical equation of kinetic energy.

$$K.E. = \frac{1}{2} m_o v^2 \qquad (6)$$

Indeed, this approximation can be generalized. All the equations of Newton mechanics can be obtained as a special case of relativistic mechanics in the limit of v<<c. In practice, it is commonly considered that the relativistic effects is negligible when v<0.2c.

In nuclear engineering applications, relativistic formulas must be used for electrons, while the errors incurred by ignoring the relativistic effects are very small for neutrons and protons. (see examples at the end of these notes)

The binding energy

So far, we know that the atom consists of a central positively changed nucleus orbiting around it negatively charged electrons. Neutral atoms contain equal number of protons and electrons. The identity of the atom is defined by the number of the positively charged protons packed in the nucleus. Since we know that like charges repel, the question is; why nuclei are (apart from hydrogen) not self destruct and force their protons apart as a result of the electrostatic repulsive forces? The answer to this question lies with the following two fundamental laws of nature:

1. The equivalence of mass and energy
2. The conservation of energy.

With these two laws, we can write the following conservation equation for the nucleus:

$$\boldsymbol{Rest\ Mass\ Energy + Kinetic\ Energy + Interna\ Excitation\ Energy = Constant}$$

In symbolic format, the above equation can be written as:

$$m_o c^2 + \frac{1}{2} m_o \text{v}^2 + Q = Constant \qquad (7)$$

We will deal with the excitation energy (Q) later. For the time being, let us disregard it. If we consider a representative nucleus of aluminum atom at its ground state (Q=0), and its kinetic energy is zero. The nucleus of ${}^{27}_{13}Al$ is made up of 14 neutrons and 13 protons. Applying equation 7 to the nucleus as a whole :

$$Rest\ mass\ of\ the\ nucleus = the\ atomic\ mass - 13(rest\ mass\ of\ electron)$$

the atomic mass of Al is 26.98154 amu, and the rest mass of electron is 0.000549 amu. Then based on the above equation, the rest mass of the nucleus is 26.9744 amu.

If we consider the rest mass of constituents of the nucleus individually; that is adding up the rest mass of 13 protons and 14 neutrons. We will get a nucleus rest mass of 27.2159 amu.

There is a **mass defect** of 0.2415, which is equivalent to $(0.2415 * 931.5 = 224.9 \text{ Mev})$

Based on this example, the rest mass of the nucleons when they are joint in a nucleus are smaller by an energy equivalent mass of 224.9 Mev. But the two laws above told us that this energy could not have disappeared. The mass of 13 protons and 14 neutrons should add up to 27.2159 of mass-energy irrespective whether they move individually or packed in a nucleus.

The question is; where did this 224.9 Mev of energy- mass equivalent go?

There is a short range (of order of 10^{-15} m) strong nuclear force between the nucleons to balance the repulsive coulomb (electrostatic) force that holds the nucleus together. That force must be stronger than Coulomb force within the nucleus boundary. If there is a binding force that holds the nucleus together, then there must be a binding energy responsible for holding the nucleus together. Indeed, the mass defect is lost when the protons and neutrons are glued together in the nucleus. This mass is changed into what is called the "***Binding Energy***" :

$$\begin{matrix} Mass\ energy\ of \\ 13p + 14n \\ individually \end{matrix} = \begin{matrix} Mass\ energy \\ 13p + 14n \\ in\ nucleus \end{matrix} + \begin{matrix} Binding \\ energy \end{matrix}$$

<u>Order of magnitude of the binding energy</u>

Let the mass defect be *Δm,* then the binding energy *BE* will be:

$$BE = \Delta m \times c^2 \qquad (8)$$

Given that the order of magnitude of the mass of a nucleon is 10^{-27} kg, then the order of magnitude of the binding energy is

$$BE = \Delta m \times 9 \times 10^{16} \times 10^{19}$$

Mind you that we are talking order of magnitude, not exact numbers

The 10^{19} is a conversion factor from joule to electron volt. So, even if a tiny bit of the rest mass of a nucleon converts to energy, the binding energy will be of order of Mega electron volt (Mev).

Calculation of the binding energy (*BE*)

Let us consider an atom of atomic number Z, mass number A, at its ground state, and of zero kinetic energy. Applying equation 7 to this atom:

$$\left(Zm_p + Nm_n + Zm_e\right)c^2 = m_{atom}({}^A_Z X)c^2 + BE_{nuc} + BE_{atom} \qquad (9)$$

For example, the electron in the hydrogen atom is bound to the atom by 13.6 ev. If one wants to take that electron off, it must be provided with at least 13.6 ev. Re-write equation 9 slightly differently

$$Z\left(m_p + m_e\right)c^2 + Nm_n c^2 = m_{atom}({}^A_Z X)c^2 + BE_{nuc} + BE_{atom} \qquad (10)$$

The bracket of the first term of equation 10 represents the rest mass of a hydrogen atom constituents, i.e.

$$\left(m_p + m_e\right)c^2 = m_{atom}({}^1_1 H)c^2 + BE_{atom-}$$

Substitute into 10 to obtain

$$Z[m_{atom}({}^1_1 H)c^2] + Nm_n c^2 + Z(BE_{atom-}\ ) = m_{atom}({}^A_Z X)c^2 + BE_{nuc} + BE_{atom} \qquad (11)$$

BE_{atom} is typically of order of magnitude of electron volts, while *BE_{nuc}* is of order of mega electron volts (6 order of magnitude bigger). It can therefore be eliminated from both sides of equation 11 without affecting the accuracy of the equation. Therefore;

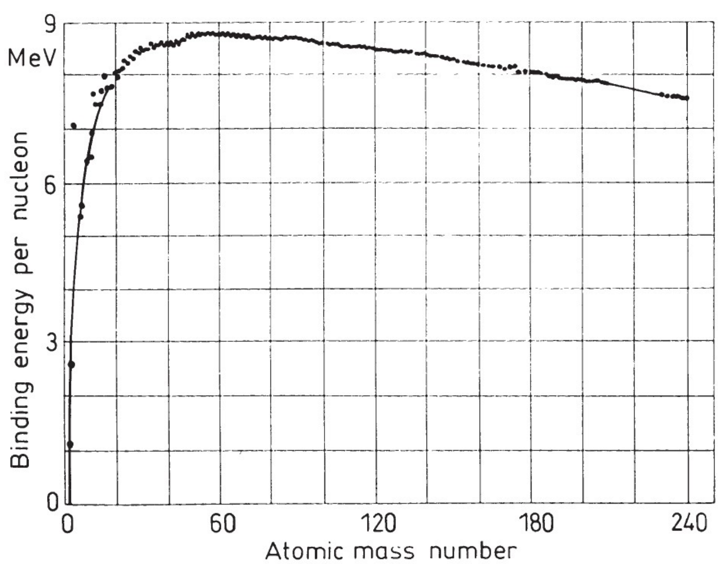
$$BE_{nuc} = Z[m_{atom}({}^1_1 H)c^2] + Nm_n c^2 - m_{atom}({}^A_Z X)c^2 \qquad (12)$$

All pieces of information on the RHS of equation 12 are known. Therefore, the binding energy (or its equivalent mass defect) can be calculated using tabulated values. The binding energy for other nuclei, namely D, Fe, and U are shown in the table below. As expected, since each nucleon adds its own contribution the total binding energy of a nucleus, nuclei with higher mass number have

Element	Total binding energy (MeV)	Average binding energy (MeV)
D	4.46	2.23
Al	234.6	8.32
Fe	453.6	8.13
U	1809.0	7.6

Total and average BE of four nuclei.

higher total binding energy. A more interesting trend is observed when we calculate the average binding energy per nucleon. It is obtained by dividing the total binding energy by the mass number. The following graph is obtained when we plot the average binding energy per nucleon against the mass number of all known isotopes.



The nuclear binding energy curve above exhibits an increasing trend of the binding energy per nucleon for the lighter nuclei (starting from A=1 up to a maximum A= 50 approximately). This trend is attributed to the overwhelming effects of the short range strong nuclear force. When the size of a nucleus is within the range of nuclear force, each added nucleon is attracted to an increasing number of nucleons. The maximum binding energy per nucleon is 8.8 Mev, which occurs at about A=50. When the size of the nucleus increases the effects of nuclear forces diminish particularly for those nucleons closer to the boundary of the nucleus. As the mass number (A) increases the influence of the electrostatic repulsive forces starts to compete with the diminishing short range nuclear forces producing a fairly flat maximum followed by slowly decreasing average binding energy per nucleon. Based on this argument, one can understand why we can't find nuclei of arbitrarily high mass number.

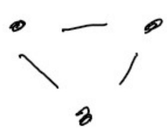
The binding energy can be thought of as an energy well. Maximum binding energy per nucleon corresponds to the deepest point in that well. Equivalently, we can say that higher average binding energy corresponds to a more stable nucleus because nucleons are bound strongly to the nucleus. Therefore, a more stable nuclei can be produced if we can split a nucleus of large mass number (say U-238) into two nuclei, each of A=119 (for example). The average binding energy per nucleon for U-238 is 7.6 Mev, while the average binding energy per nucleon for A=119 is about 8.5 Mev. That is 0.9 Mev is released per nucleon or 214 Mev for each nucleus split. That is what happens during the fission reaction in a nuclear reactor.

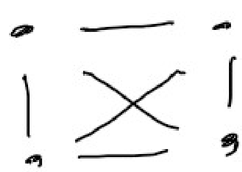
Phenomenological Formula to Calculate the binding energy

Initially it was believed that the nuclear force which holds the nucleus together is acting between each pair of nucleons

p-p, n-n, or n-p

A

3  3 pairs

4  6 pair

In general for any A, the number of pairs is given by $\frac{A^2-A}{2}$. Since each pair contributes to the total binding energy then the binding energy will be proportional to $\frac{A^2-A}{2}$.

Roughly $BE \propto A^2$ and $\frac{BE}{A} = \alpha A$ where α is called the volume constant. However, the curve of *BE/A* against *A*, as shown above, is not linear. Therefore, Many corrections are needed to account for other factors affecting the binding energy. The following equation was proposed to calculate the binding energy:

$$BE = \alpha A - \beta A^{2/3} - \frac{\gamma Z^2}{A^{\frac{1}{3}}} - \frac{\zeta(A - 2Z^2)}{A} + \delta \qquad (13)$$

The value of the constants in Mev are as follow:

α=15.56, β= 17.23, γ=0.697, ζ=23.285, and δ=±12.0. δ is negative if both N and Z are odd, positive if both are even and zero if either N or Z is odd and the other is even.

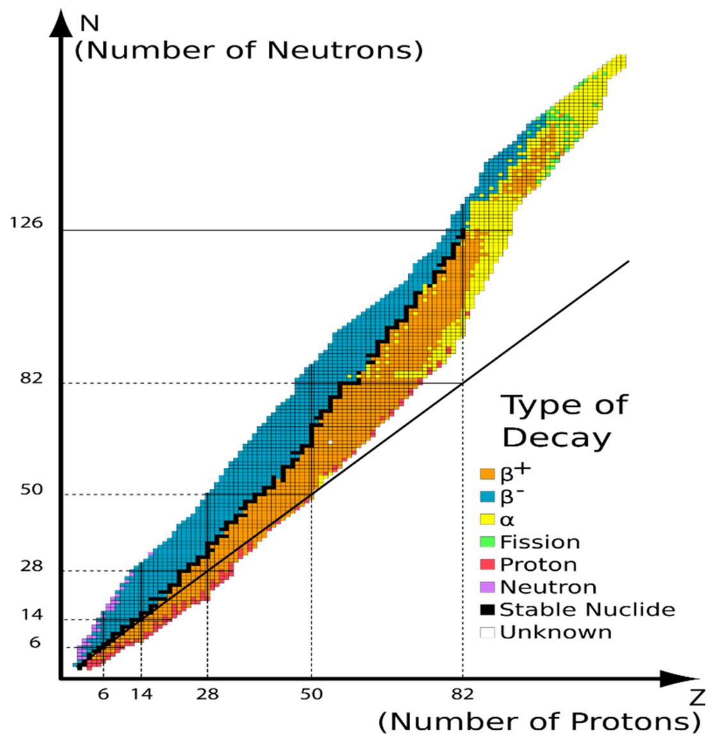
The physical meaning of the terms on the RHS of equation 13 will now be discussed term by term. Seeing that the attracting forces between the nucleons mainly operate via direct neighbours, the positive first term in (13) will be proportional to the number of nucleons A in the nucleus. One can call this the volume term. Hereby it is neglected, however, that particles at the surface of the nucleus are not completely surrounded by other particles. Consequently, the binding energy has been overestimated with an amount that must be proportional to the surface area of the nucleus. By analogy with a liquid drop this effect is indicated as the surface tension effect. This term (surface term) must be proportional to the surface area, so with $A^{2/3}$. The third term is connected with the coulomb interaction between the protons, which lowers the binding energy because of the repulsion between charges of equal sign. The potential energy of an electrically charged sphere is proportional to Z^2/R, whereby R is proportional to $A^{1/3}$, which yields the third term (coulomb term). The last two terms in (13) cannot be described as 'classically' as the first three. In atomic nuclei there is a tendency to form groups of neutron/proton pairs. Especially in the case of light nuclei, one sees that those with an equal number of protons and neutrons are very stable. The heavier stable nuclei, however, contain more neutrons than protons. This excess of neutrons is necessary in order that the attractive forces between the neutrons and between the neutrons and protons can provide some compensation for the repulsion between the protons (third term). At the same time, however, some instability is introduced because the surplus of neutrons occupies a number of energy levels in the nucleus, which do not contain protons. A correction factor must be introduced for this, the so-called symmetry term, which is also important in the case of a proton surplus; therefore this term is quadratic in N-Z.

The last term is the pairing term, which accounts for the fact that nucleons have a spin moment of momentum or 'spin'. This spin effect finds expression in the fact that nuclei with an even number of protons and an even number of neutrons are very stable thanks to the occurrence of 'paired spin'. When a nucleus contains an odd number of both particle types, it is nearly always Unstable.

Nuclear Stability

The state of lowest energy at which an atom is normally found is called "Ground State". when an atom possesses more energy than that of ground state, it is said to be in excited state or at an energy level. An atom can not stay at excited state indefinitely... it decays to one or another lower energy level and eventually to the ground state. During such transition a photon is emitted with energy equal to the difference between the two states. Like atoms, nuclei can be excited and can occupy discrete energy levels. Nuclear Energy is much larger than that of the atom due to the fact that nuclear forces that hold the nucleus together are about 5 orders of magnitude larger than the atomic forces.

There are more than 3000 known isotopes, about 200 of them are naturally occurring. The rest are generated artificially. If we plot for all known isotopes a Z vs N graph, we will obtain the figure below.



The black dots represent the stable nuclides. The colored dotes above and below the stable (black) dotes are unstable and will decay eventually to a stable state. At the lower end of the plot one will find that stable nuclides have equal number of protons and neutrons (up to N=20, A=40). After that, the stability line diverts toward the neutron end, i.e. nuclei need N> Z to be stable. The ratio N/Z increases roughly from 1 to 1.4 as we go up in the mass number. One will also find unstable (colored) nuclides above and below the stable ones. we can also note that all possible nuclides are contained inside an envelope. we can not find a nuclide below, above or beyond that envelope.

The question now is; why is it some nuclei are stable others unstable and other combinations of N and Z do not exist at all?

Let us first consider what happen with these unstable nuclei. Basically, for the nuclei above the stability line, they got access neutrons and for those below the stability line, they got unstable nuclei with excess protons. Unstable nuclei (above and below the stability line) are going to change (decay) spontaneously to get back to stability conditions. Consider first a circle above the stability line which means it has excess neutrons. A neutron will decay to a proton plus one electron plus antielectron neutrino.

$$n \rightarrow p^{+} + e^{-} + \overline{v}$$

If on the other hand we have access of protons, then a proton will convert into a neutron plus a positron and a positron neutrino

$$p^{+} \rightarrow n + e^{+} + v$$

What is happening is that if a nucleus is above the stability line, it got too many neutrons. Through the decay process it is going to decrease the neutrons by one and increase the protons by one and it is going to emit an electron. This is called β decay.

if you get to the very high end of the graph (N>80) you will not get electron emission (β decay) but will most likely will got emission of α particles. heavy nucleus will decrease neutrons by two and protons by two to move toward stability line

It is important to mention at this stage that so far, we discussed what happens when a nucleus is unstable, but we did not say why? The complete answer to this question is beyond the scope of this course.

<u>Principles of nuclear reaction:</u>

It is defined as an interaction between nuclear particles that results in changing the identity or the characteristics of the particles involved. That include nucleus, neutron, photon, radioactive decay, …. etc. Of particular interest to this course, is the fission reaction, in which a heavy nucleus interacts with a neutron. A nuclear reaction can be represented by

$$a + b \rightarrow c + d \qquad (14)$$

The fundamental laws governing this reaction are;

1. Conservation of energy

2. Conservation of momentum
3. Conservation of charge
4. Conservation of the number of nucleons

Item 4 does not mean that protons and neutrons are conserved separately. one should also keep in mind that it is limited to nuclear particles (e.g. electrons are not considered for conservation.

The conservation of energy for equation 14, considering the rest mass *m* and the *K.E.* of interacting entities is

$$(K.E._a + m_a c^2) + (K.E._b + m_b c^2) = (K.E._c + m_c c^2) + (K.E._d + m_d c^2)$$

Re-arranging the last equation

$$(K.E._c + K.E._d) - (K.E._a + K.E._b) = [(m_a + m_b) - (m_c + m_d)]c^2 \qquad (15)$$

If the RHS of equation 15 is positive, which means that a bit of the mass has converted to energy, then such nuclear reaction is said to be exothermic. if it is negative then it is said to be endothermic; that is K.E. has converted to mass. When the masses are given in amu's then the energy generated/consumed in Mev's is given by

$$[(m_a + m_b) - (m_c + m_d)] \times 931.5$$

As, we have already shown that,

1 amu= 931.5 Mev

In nuclear engineering, we are mostly interested in the interaction between nuclear particles or photons with a nucleus. A typical nuclear reaction is generally achieved by firing nuclear particles (say b) on a target nucleus (say a), then the nuclear reaction (14) can be abbreviately written as **a(b, c) d** or **a(b, d)c** whichever more convenient.
For example, the reaction

$$^{16}O + n \rightarrow {}^{16}N + {}^{1}H$$

can be written in abbreviated form as

$$^{16}O(n,p)^{16}N$$

where *p* is a proton representing a hydrogen nucleus.

Examples

1. What is the energy equivalent of: (a) the electron rest mass, (b) the amu.
2. In fission reactors one deals with neutrons having kinetic energies as high as 10 MeV. How much error is incurred in computing the speed of 10-MeV neutron by using the classical expression rather than the relativistic expression.
3. A person in north America typically consumes 10^{11} joules annually. (a) if the combustion of a CH_2 molecule produces about 6.242 ev, calculate the mass of CH_2 required to provide that amount of energy, (b) if the fission of a U-235 produces 200 Mev, calculate the mass of uranium required to supply that amount of energy.
4. Calculate the mass and binding energy of ${}^{107}_{47}Ag$ using the empirical (phenomenological) equation.
5. Calculate the binding energy of the last neutron of ${}^{16}_{8}O$. This is the energy released in the nuclear reaction

   $${}^{15}_{8}O + {}^{1}_{0}n \rightarrow {}^{16}_{8}O$$

   Given that M(${}^{15}_{8}O$) = 15.0030654, M(${}^{16}_{8}O$) = 15.9949146. (All in atomic mass units u).

6. Based on the conservation principles, complete the following reactions and calculate the energy generated/consumed from then reaction:
   1. ${}^{4}_{2}He(p, {}^{2}_{1}H)?$, 2. ${}^{9}Be(\propto, n)?$, 3. ${}^{14}N(n,p)?$, 4. ${}^{115}In({}^{2}H, p)?$, 5. ${}^{207}Pb(\gamma, n)?$

7. The isotope thorium-232 (${}^{232}_{90}Th$) decays successively to form ${}^{228}_{88}Ra$, ${}^{228}_{89}Ac$, ${}^{228}_{90}Th$, and ${}^{224}_{88}Ra$, and finally becoming radon-220 (${}^{220}_{86}Rn$)What particles are emitted in each of these five steps?